The Random Graph

Step 1. $R \subseteq \mathbb{N} \times \mathbb{N}$ is a *dense linear order* on $\mathbb{N}$ if the following axioms are satisfied.

(1) xRy implies $x \neq y$.
(2) xRy or yRx.
(3) xRy and yRz implies xRz.
(4) xRy implies there is a z so that xRz and zRy.

Here xRy means $(x, y) \in R$.

Two dense linear orders R and $\tilde{R}$ are *isomorphic* if there is a bijection $b : \mathbb{N} \to \mathbb{N}$ such that

$$xRy \text{ if and only if } b(x)\tilde{R}b(y).$$

Up to isomorphism, how many dense linear orders are there on $\mathbb{N}$?

Step 2. Construct and infinite undirected graph on $\mathbb{N}$ as follows: for each pair $a, b \in \mathbb{N}$ with $a < b$, flip a fair coin and place an edge between a and b if the toss comes up heads. Given two graphs constructed in this way, what is the probability they are isomorphic?