Graph Mixing

Recall that a *finite directed graph* consists of a finite set of *vertices* V and a set of *edges* $E \subseteq V \times V$. When $(a, b) \in E$, we say that b is a *neighbor* of a and we define

$$N(a) \equiv^{def} \{b \in V : (a, b) \in W\}.$$

the set of neighbors of a.

A *random walk* on $G = (V, E)$ is a sequence of vertices $a_0, a_1, a_2, ... \in V$ where each a_{i+1} is chosen uniformly at random from $N(a_i)$.

Given a fixed starting point $a_0 \in V$ and $b \in V$, we'd like to understand

$$P(a_n = b) \text{ as } n \to \infty.$$

Consider the following two (undirected) examples first. The length n line

$$V = \{1, ..., n\} \text{ with } E = \{(a, b) \in V : |a - b| = 1\},$$

and the length n circle

$$V = \{1, ..., n\} \text{ with } E = \{(a, b) \in V : |a - b| \equiv 1 (\text{mod } n)\}.$$

Can you generalize to arbitrary directed graphs?