

Line Losses

Suppose electrons are moving through a wire. Each electron moves either to the left or to the right with velocity 1. After a time step Δt , each electron changes direction with probability $p \in (0, 1)$.

Suppose you have a wire of length L and you feed right-moving electrons into the left endpoint at a constant rate C . On average, what is the rate of electrons leaving the right endpoint?

A continuous version in an infinite wire might work as follows. Let $u(x, t)$ and $v(x, t)$ be the density of right-moving and left-moving electrons, respectively at position x and time t . u and v satisfy the pair of partial differential equations

$$(1) \quad \frac{\partial u}{\partial t}(x, t) + \frac{\partial u}{\partial x}(x, t) = p(v(x, t) - u(x, t)),$$

$$(2) \quad \frac{\partial v}{\partial t}(x, t) - \frac{\partial v}{\partial x}(x, t) = p(u(x, t) - v(x, t)).$$

For a few choices of initial data $u(x, 0)$ and $v(x, 0)$, describe what happens as $t \rightarrow \infty$.