Knots

A *knot* is a smooth embedding of the circle into three dimensional space. That is, a smooth injection

$$K : S^1 \to \mathbb{R}^3,$$

where S^1 denotes the set $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$.

Two knots K_1 and K_2 are *equivalent* if there is a smooth function $h : S^1 \times [0, 1] \to \mathbb{R}^3$ such that

(1) $h(-, t)$ is a knot for each $t \in (0, 1)$.
(2) $h(-, 0) \equiv K_1$ and $h(-, 1) \equiv K_2$.

where $h(-, t) : S^1 \to \mathbb{R}^3$ denotes the function $x \mapsto h(x, t)$.

We could instead project into $\mathbb{R}^2$, if we are willing to handle crossings. What is the correct definition? What is the analog of equivalence?

Given the proper definition, one can consider colorings of a 2D projection: to each arc, we assign a color from $\{red, green, blue\}$ such that the three arcs meeting at any given crossing have all same or all different colors. A coloring is non-trivial if it uses more than one color.

Show that the property "K admits a non-trivial coloring" is preserved by equivalence. Use this to show some knots are not equivalent.

Where else can you go with these ideas?