Math 128A, Spring 2007

Programming Assignment 1 Solution

The answers to the numbered questions are here. The code to generate the tables and plots can be found in a separate file.

1. I ran my secant method implementation with

$$f(x) = \sin(x)/x^2 - \cos(x)/x$$

and generated the following data

	run 1	run 2	run 3
p_0	3.0000	7.0000	1.0000e-01
p_1	3.1000	7.1000	9.0000e-01
p_2	4.8147	7.7642	-9.5880e-03
p_3	4.5333	7.7262	8.0948e-04
p_4	4.4898	7.7252	-6.8134e-09
p_5	4.4934	7.7253	
p_6	4.4934	7.7253	
p_7	4.4934		
p_8	4.4934		

The first two positive roots of $f(x)$ are displayed. When the iteration starts close to zero, it converges to zero. This is expected, since the above formulation of $f(x)$ can be extended by letting $f(0) = 0$, yielding a continuously differentiable function with a simple root at 0.

2. Since

$$p(x) = x^5 - 3^4 + 2x^3 + 2x^2 - 3x + 1 = (x-1)^4(x+1)$$

the roots are ± 1. Here is the data from two runs of the secant method converging to the two roots:

i	p_i	ϵ_i	$\epsilon_{i+1}/\epsilon_i$
0	0.5000	5.0000e-01	8.0000e-01
1	0.6000	4.0000e-01	8.0602e-01
2	0.6776	3.2241e-01	8.0895e-01
3	0.7392	2.6081e-01	8.1143e-01
4	0.7884	2.1163e-01	8.1308e-01
5	0.8279	1.7207e-01	8.1438e-01
6	0.8599	1.4013e-01	8.1536e-01
⋮	⋮	⋮	⋮
41	0.9999	1.2709e-04	8.1917e-01
42	0.9999	1.0411e-04	8.1917e-01
43	0.9999	8.5280e-05	

and

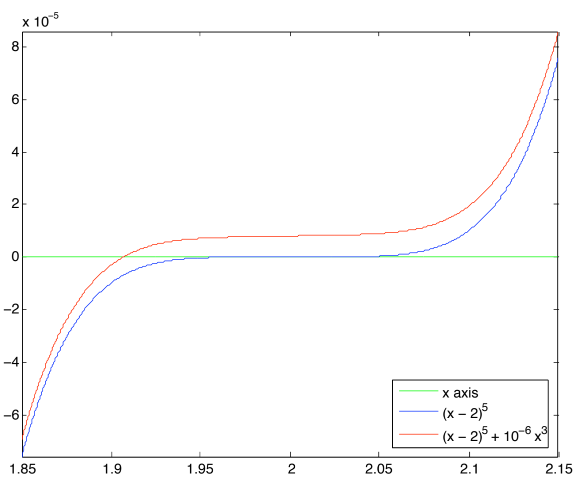
i	p_i	ϵ_i	$\epsilon_{i+1}/\epsilon_i^{\phi}$
0	-1.5000	5.0000e-01	1.2278e+00
1	-1.4000	4.0000e-01	8.2806e-01
2	-1.1880	1.8801e-01	1.2862e+00
3	-1.0861	8.6083e-02	1.2721e+00
4	-1.0241	2.4054e-02	1.5134e+00
5	-1.0036	3.6365e-03	1.4962e+00
6	-1.0002	1.6907e-04	1.5518e+00
7	-1.0000	1.2238e-06	1.5241e+00
8	-1.0000	4.1375e-10	1.6899e+00
9	-1.0000	1.1102e-15	0.0000e+00
10	-1.0000	0.0000	

where $\phi = (1+\sqrt{5})/2$

According to the data, the secant method converges linearly to 1 and with rate ϕ to -1 (note the near constant ratios in the third columns). This is exactly what was expected, given that 1 is a multiple root while -1 is simple.

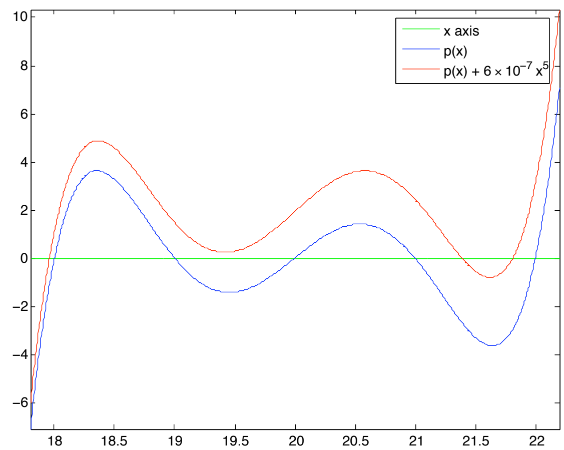
The output for -1 is more accurate (in fact, exact in floating-point). This is because $f(x)$ is very flat near 1, so the criterion $|f(p_i)| < ytol$ terminates the iteration early.

3. In the neighborhood of $x = 2$, adding $10^{-6}x^3$ to $p(x) = (x-2)^5$ has about the same effect as adding 2×10^{-6}. Thus we expect the new polynomial to have exactly one real root, which is simple, slightly to the left of 2. Here is a graph of the situation:



The secant method computes the new root as $r = 1.9071$. The ratio of change in root to change in coefficient is 9.2943×10^4, which is enormous.

4. As in the previous problem, it's easy to see what's going on when the graph is in front of you.



We see that the roots 19 and 20 become a pair of complex conjugate roots, while the other three shift around. 18, 21, and 22 shift to 17.9572, 21.3913, and 21.8123, respectively, with relative changes 0.0024, 0.0186, and 0.0277, respectively.

It is important to notice that the relative changes of the roots are much larger than the relative changes of the polynomial coefficients: the coefficient of x^5 experiences a relative change of only 1.8987×10^{-13}, while the others are unchanged.