Problem written on board

M128A, HW9

$$y'(t) = t^2 - y(t)^2, \quad y(0) = 1$$

by Taylor series method of local order h^3 to get $y(0.2)$, $h = 0.1$.

$$f(x, z) = x^2 - z^2$$

$$Y(x_{n+1}) \approx Y(x_n) + h Y'(x_n) + \frac{h^2}{2} Y''(x_n) + \frac{h^3}{6} Y'''(x_n)$$

$$Y'(t) = t^2 - Y(t)^2$$

$$Y''(t) = 2t - 2Y(t)Y'(t)$$

$$= 2t - 2Y(t)(t^2 - Y(t)^2)$$

$$= 2t - 2t^2 Y(t) + 2Y(t)^3$$

$$Y'''(t) = 2 - 4t Y(t) - 2t^2 Y'(t) + 6 Y(t)^2 Y'(t)$$

$$= \cancel{2 - 4t}$$

$$\cancel{2 Y(t)[2t}$$

$$= 2 - 4t Y(t) - 2t^2(t^2 - Y(t)^2) + 6 Y(t)^2 (t^2 - Y(t)^2)$$

$$= 2 - 4t Y(t) - 2t^4 + 2t^2 Y(t)^2 + 6t^2 Y(t)^2 - 6 Y(t)^4$$

$$= 2 - 4t Y(t) - 2t^4 + 8t^2 Y(t)^2 - 6 Y(t)^4$$

$$y_{n+1} = y_n + h\left(x_n^2 - y_n^2\right) + \frac{h^2}{2}\left(2x_n - 2x_n^2 y_n + 2y_n^3\right) + \frac{h^3}{6}\left(2 - 4x_n y_n - 4x_n^4 + 8x_n^2 y_n^2 - 6y_n^4\right)$$

$x_0 = 0 \qquad y_0 = 1$

$x_1 = h = 0.1 \qquad y_1 = .909\overline{333}$

$x_2 = 0.2 \qquad y_2 = .8356728097$

M128A, HW9

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$\boxed{\gamma_2 = \frac{3}{4}}$ ~~or $\gamma_2 = 1$~~

gives by page 412

$1 - \gamma_1 - \gamma_2 = 0$

so $\boxed{\gamma_1 = \frac{1}{4}}$

$1 - 2\gamma_2 \alpha = 0$

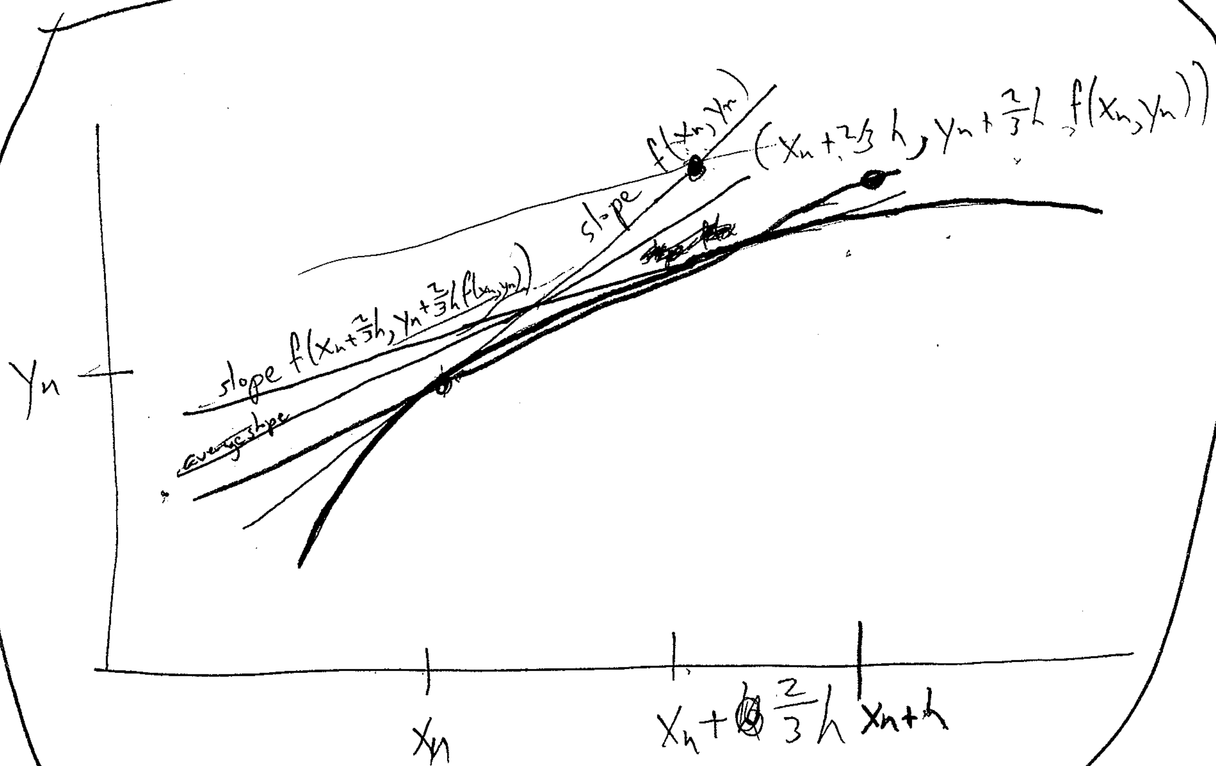
so $\boxed{\alpha = \beta = \frac{2}{3}}$

p 422 #6

So for $a_2 = \frac{3}{4}$ we get

$$y_{n+1} = y_n + h\,F(x_n, y_n; h)$$

$$F(x_n, y_n; h) = \frac{1}{4} f(x_n, y_n) + \frac{3}{4} f\left(x_n + \frac{2}{3}h,\ y_n + \frac{2}{3}h f(x_n, y_n)\right)$$



~~don't do this~~ Do it

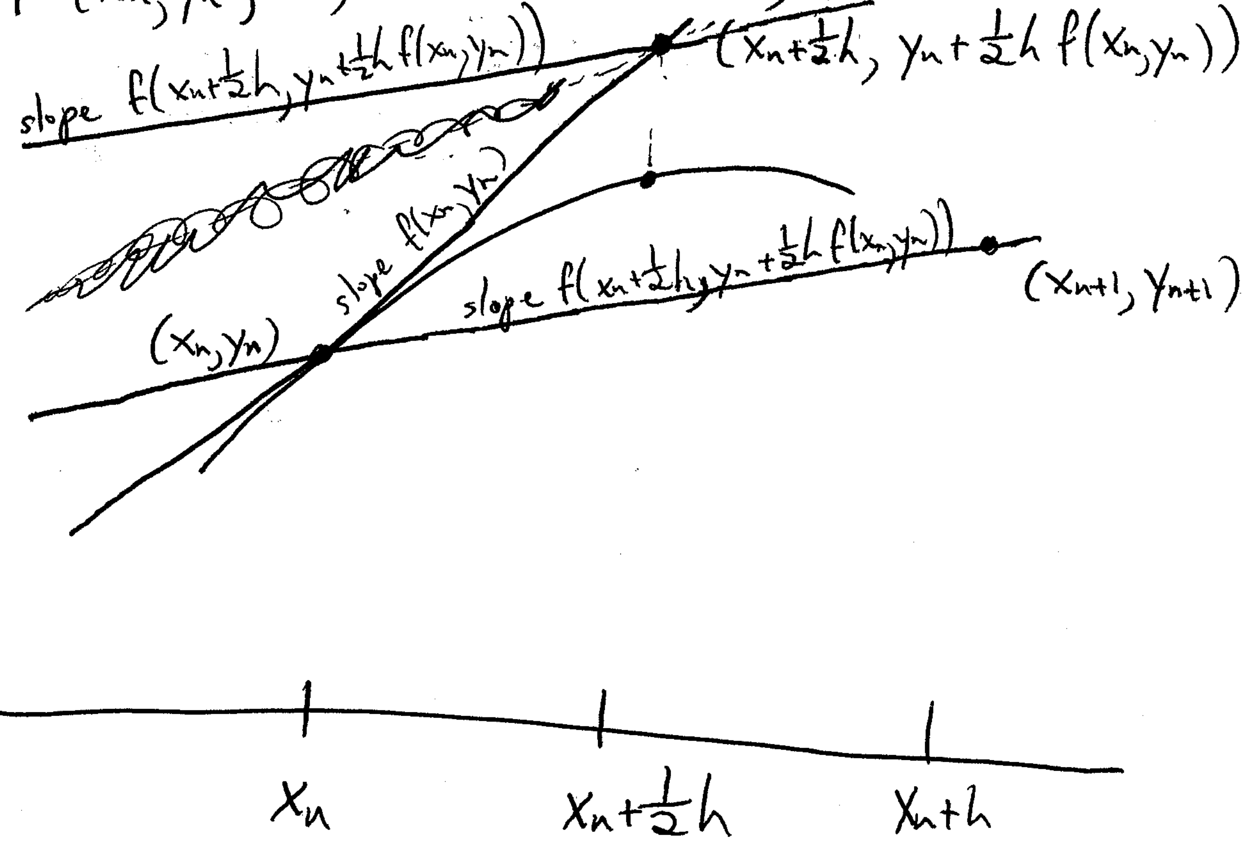
p. 422 #6

For $\gamma_2 = 1$ we get $\gamma_1 = 0$

and $\alpha = \beta = \frac{1}{2}$, so

$y_{n+1} = y_n + h\,F(x_n, y_n; h)$ where

$F(x_n, y_n; h) = f\left(x_n + \frac{1}{2}h,\ y_n + \frac{h}{2} f(x_n, y_n)\right)$



Actual Answer to the IVP

$y'(t) = t^2 - y(t)^2, \quad y(0) = 1, \quad h = 0.1$

```
> ?ode
> ode := diff(y(x),x) = x^2 - y(x)^2;
```

$$ode := \frac{d}{dx}\,\mathrm{y}(x) = x^2 - \mathrm{y}(x)^2$$

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> ics := y(0)=1;
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$$ics := \mathrm{y}(0) = 1$$

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> evalf(subs(x=0.2, dsolve({ode,ics})),400);
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y(0.2) = 0.83578501750571659617529482681560922181349804927065075337646502034 81\
8900939625925292637061421247333410446800225064586645770622155860882126 71502\
5290782365273473630826897458403620594753862233546384467327053517016749 67187\
9203151609796884073376891836889992307887083619003100735805271256284898 35371\
8526120965553363414019306732692650136667357697016420536173990376636558 96504\
06380637144728629695459329852 6992

```
>
```

Runge Kutta fourth order

$$A = V_1 = f(x_n, y_n) = x_n^2 - y_n^2$$

$$B = V_2 = f\left(x_n + \tfrac{1}{2}h,\ y_n + \tfrac{1}{2}hV_1\right)$$
$$= \left(x_n + \tfrac{1}{2}h\right)^2 - \left(y_n + \tfrac{1}{2}hV_1\right)^2$$

$$C = V_3 = f\left(x_n + \tfrac{1}{2}h,\ y_n + \tfrac{1}{2}V_2\right)$$
$$= \left(x_n + \tfrac{1}{2}h\right)^2 - \left(y_n + \tfrac{1}{2}hV_2\right)^2$$

$$D = V_4 = f\left(x_n + h,\ y_n + hV_3\right)$$
$$= (x_n + h)^2 - (y_n + hV_3)^2$$

$$y_{n+1} = y_n + \frac{h}{6}\left[V_1 + 2V_2 + 3V_3 + V_4\right]$$

$x_0 = 0$ $\qquad y_0 = 1$

$x_1 = 0.2$ $\qquad y_1 = .8357925801$

Apply <u>modified Euler method</u> and ~~Runge Kutta~~ ~~methods of global error h^2~~

<u>Heun's method</u>, both Runge-Kutta methods of global error h^2, on

$$y'(t) = t^2 - y(t)^2, \quad y(0) = 1, \quad h = 0.1$$

~~Modified Euler method~~ <u>Heun's Method (8.81)</u>

$$y_{n+1} = y_n + \frac{h}{2}\left[f(x_n, y_n) + f(x_n + h, y_n + h f(x_n, y_n))\right]$$

$$y_{n+1} = y_n + \frac{0.1}{2}\left[x_n^2 - y_n^2 + (x_n + 0.1)^2 - (y_n + 0.1(x_n^2 - y_n^2))^2\right]$$

$x_1 = 0.1 \quad y_1 = .91$

$x_2 = 0.2 \quad y_2 = .836800066195$ ✓

Modified Euler's Method for $y'(t) = t^2 - y(t)^2$

$$y_{n+1} = y_n + h\, f\left(x_n + \tfrac{1}{2}h,\ y_n + \tfrac{1}{2}h\, f(x_n, y_n)\right)$$

$h = 0.1$

$x_0 = 0 \quad y_0 = 1$

$x_1 = 0.1$

$$y_{n+1} = y_n + 0.1\left[\left(x_n + \tfrac{1}{2}h\right)^2 - \left(y_n + \tfrac{1}{2}h\left(x_n^2 - y_n^2\right)\right)^2\right]$$

$x_1 = 0.1 \qquad y_1 = .91$

$x_2 = 0.2 \qquad y_2 = .8367173880975$ ✓