

Problem written on board M128A, HW9

$$y'(t) = t^2 - y(t)^2, \quad y(0) = 1$$

by Taylor series method of local order h^3 to get $y(0.2)$, $h = 0.1$.

$$f(x, z) = x^2 - z^2$$

$$Y(x_{n+1}) \approx Y(x_n) + h Y'(x_n) + \frac{h^2}{2} Y''(x_n) + \frac{h^3}{6} Y'''(x_n)$$

$$Y'(t) = t^2 - Y(t)^2$$

$$Y''(t) = 2t - 2Y(t)Y'(t)$$

$$= 2t - 2Y(t)(t^2 - Y(t)^2)$$

$$= 2t - 2t^2 Y(t) + 2Y(t)^3$$

$$Y'''(t) = 2 - 4t Y'(t) - 2t^2 Y''(t)$$

$$+ 6Y(t)^2 Y'(t)$$

$$= 2 - 4t$$

$$2 - 4t Y(t) + 6Y(t)^2 (t^2 - Y(t)^2)$$

$$\begin{aligned} &= 2 - 4t Y(t) - 2t^2 (t^2 - Y(t)^2) + 6Y(t)^2 (t^2 - Y(t)^2) \\ &= 2 - 4t Y(t) - 2t^4 + 2t^2 Y(t)^2 + 6t^2 Y(t)^2 - 6Y(t)^4 \\ &= 2 - 4t Y(t) - 2t^4 + 8t^2 Y(t)^2 - 6Y(t)^4 \end{aligned}$$

Printed on the reverse side

$$Y''(x) - \lambda Y(x) = 0$$

where $\lambda = -\mu^2$ or $\lambda = \mu^2$

$$1.0 = \mu^2 \Rightarrow \mu = 1$$

$$Y''(x) - Y(x) = 0$$

$$Y(x) = C_1 e^x + C_2 e^{-x}$$

$$Y'(x) = C_1 e^x - C_2 e^{-x}$$

$$Y(0) = C_1 + C_2 = 1$$

$$Y'(0) = C_1 - C_2 = 0$$

$$C_1 = C_2 = \frac{1}{2}$$

$$Y(x) = \frac{1}{2}(e^x + e^{-x})$$

$$Y(x) = \cosh(x)$$

$$Y''(x) - Y(x) = 0$$

$$Y(x) = \cosh(x)$$

$$\begin{aligned}
 y_{n+1} &= y_n + h(x_n^2 - y_n^2) \\
 &\quad + \frac{h^2}{2}(2x_n - 2x_n^2 y_n + 2y_n^3) \\
 &\quad + \frac{h^3}{6}(2 - 4x_n y_n - 4x_n^4 + 8x_n^2 y_n^2 - 6y_n^4)
 \end{aligned}$$

$$x_0 = 0 \quad y_0 = 1$$

$$x_1 = h = 0.1$$

$$x_2 = 0.2$$

$$y_1 = \del{0.8660} .909333$$

$$y_2 = \del{.8346584550}$$

$$.8356728097$$

$$(a^2 - b^2) \div (a - b) = a + b$$

$$(a^2 + ab + b^2) \div (a + b) = a + \frac{b^2}{a + b}$$

$$(a^2 - ab + b^2) \div (a - b) = a + \frac{b^2}{a - b}$$

$$x^2 = a^2 \quad a = x$$

IEE P.P.P. ~~1000~~ = 1000 1.0 = 1 = 1000

~~1000000~~ = 1000000 1000 = 1000

Y.P.O.S. 6.F.2.58.

p. 422 #6

~~$\lambda_1 = \frac{3}{4}$~~ $\lambda_2 = \frac{3}{4}$ or $\lambda_2 = \frac{3}{4}$

gives by page 412

$$1 - \lambda_1 - \lambda_2 = 0$$

so $\lambda_1 = \frac{1}{4}$

$$1 - 2\lambda_2 \alpha = 0$$

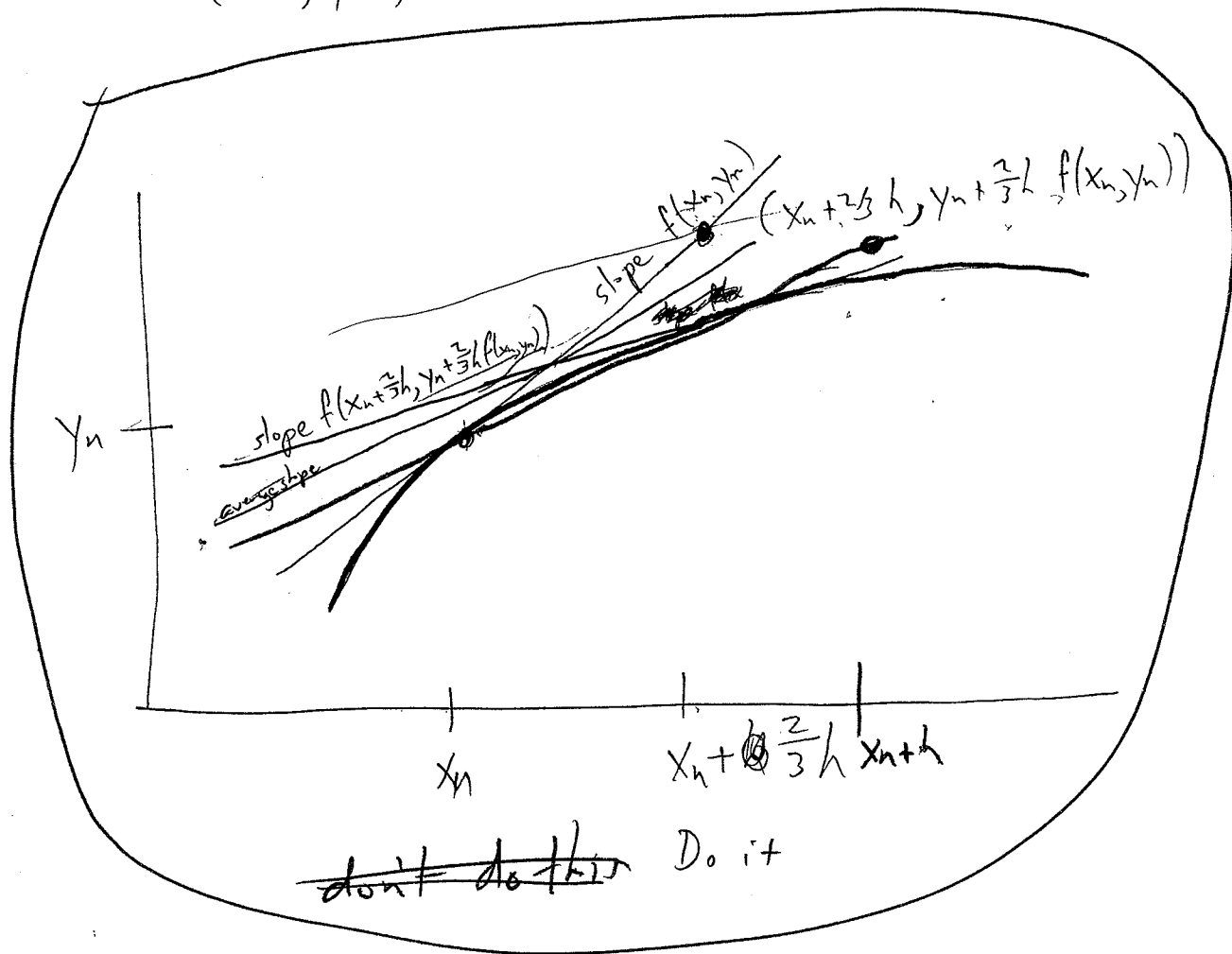
so $\alpha = \frac{2}{3}$
 $\beta = \frac{2}{3}$



P 422 #6
 So for $\alpha_2 = \frac{3}{4}$ we get

$$Y_{n+1} = Y_n + h F(x_n, Y_n; h)$$

$$F(x_n, Y_n; h) = \frac{1}{4} f(x_n, Y_n) + \frac{3}{4} f\left(x_n + \frac{2}{3}h, Y_n + \frac{2}{3}hf(x_n, Y_n)\right)$$





p. 422 #6 ~~100~~

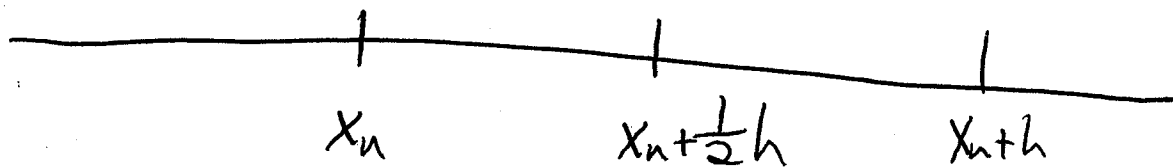
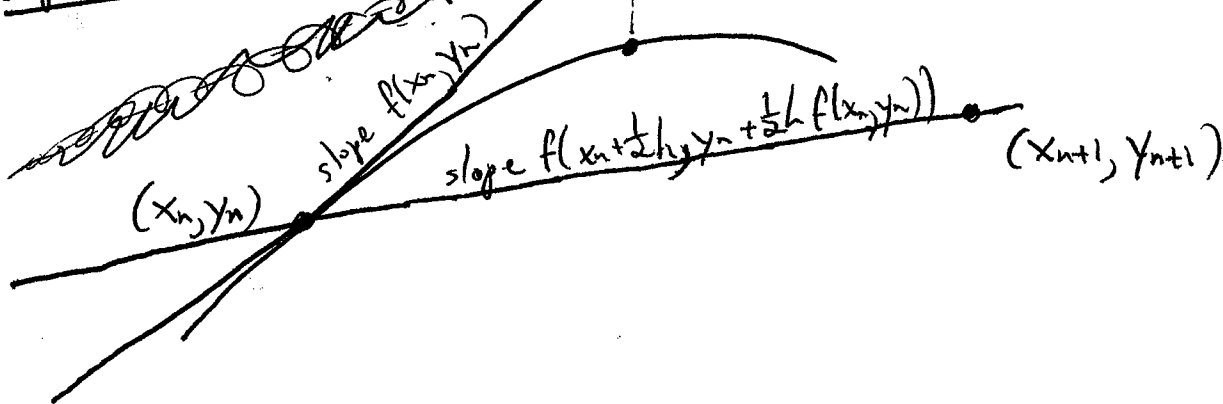
For $\gamma_2 = 1$ we get $\gamma_1 = 0$

and $\alpha = \beta = \frac{1}{2}$, so

$y_{n+1} = y_n + h F(x_n, y_n; h)$ where

$$F(x_n, y_n; h) = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}h f(x_n, y_n)\right)$$

slope $f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}h f(x_n, y_n)\right)$ $\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}h f(x_n, y_n)\right)$

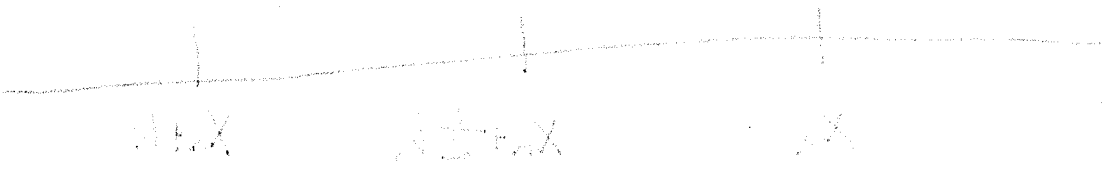
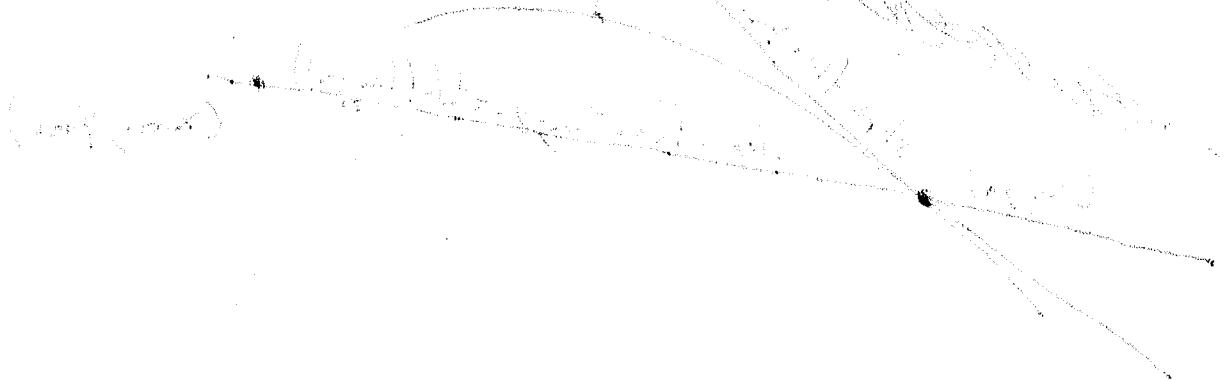


For $S_0 = 1$ we get $\theta = 0$

and $\sigma = \frac{1}{6} - \theta = \frac{1}{6}$

and $\gamma = \frac{1}{2} - \theta = \frac{1}{2}$

$f(x) = \frac{1}{\sigma} \exp\left(-\frac{x^2}{2\sigma^2}\right) = \frac{1}{\frac{1}{6}} \exp\left(-\frac{x^2}{2 \cdot \frac{1}{36}}\right) = 6 \exp\left(-\frac{18x^2}{2}\right) = 6 \exp(-9x^2)$



Actual Answer to the IVP

$$y'(t) = t^2 - y(t)^2, \quad y(0) = 1, \quad h = 0.1$$

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[ > ?ode
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[ > ode := diff(y(x), x) = x^2 - y(x)^2;
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$$ode := \frac{d}{dx} y(x) = x^2 - y(x)^2$$

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[ > ics := y(0)=1;
```

$$ics := y(0) = 1$$

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[ > evalf(subs(x=0.2, dsolve({ode, ics})), 400);
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y(0.2) = 0.8357850175057165961752948268156092218134980492706507533764650203481\  
890093962592529263706142124733341044680022506458664577062215586088212671502\  
529078236527347363082689745840362059475386223354638446732705351701674967187\  
920315160979688407337689183688999230788708361900310073580527125628489835371\  
852612096555336341401930673269265013666735769701642053617399037663655896504\  
063806371447286296954593298526992
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THE STATE OF NEW YORK
IN SENATE

January 10, 1911.
REPORT
OF THE
COMMISSIONERS OF THE LAND OFFICE

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Runge Kutta fourth order

$$A = V_1 = f(x_n, y_n) = x_n^2 - y_n^2$$

$$B = V_2 = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}hV_1\right) \\ = (x_n + \frac{1}{2}h)^2 - (y_n + \frac{1}{2}hV_1)^2$$

$$C = V_3 = f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}V_2\right) \\ = (x_n + \frac{1}{2}h)^2 - (y_n + \frac{1}{2}hV_2)^2$$

$$D = V_4 = f(x_n + h, y_n + hV_3) \\ = (x_n + h)^2 - (y_n + hV_3)^2$$

$$y_{n+1} = y_n + \frac{h}{6} [V_1 + 2V_2 + 3V_3 + V_4]$$

$$\Rightarrow x_0 = 0 \quad y_0 = 1$$

$$x_1 = 0.2 \quad \text{~~0.2~~$$

$$y_1 = \text{~~0.8357925801~~} \quad \checkmark \\ .8357925801$$

Probability

$$A = \frac{1}{2} + \frac{1}{2} = 1$$

$$B = \frac{1}{2} + \frac{1}{2} = 1$$

$$C = \frac{1}{2} + \frac{1}{2} = 1$$

$$D = \frac{1}{2} + \frac{1}{2} = 1$$

$$E = \frac{1}{2} + \frac{1}{2} = 1$$

$$F = \frac{1}{2} + \frac{1}{2} = 1$$

$$G = \frac{1}{2} + \frac{1}{2} = 1$$

$$H = \frac{1}{2} + \frac{1}{2} = 1$$

$$I = \frac{1}{2} + \frac{1}{2} = 1$$

$$J = \frac{1}{2} + \frac{1}{2} = 1$$

108269

Apply modified Euler method and

~~Runge-Kutta method~~ ~~global error~~

Heun's method, both Runge-Kutta methods of

global error h^2 , on

$$y'(t) = t^2 - y(t)^2, y(0) = 1, h = 0.1$$

~~Modified Euler method~~

Heun's Method (8.81)

$$y_{n+1} = y_n + \frac{h}{2} [f(x_n, y_n) + f(x_n+h, y_n + h f(x_n, y_n))]$$

$$y_{n+1} = y_n + \frac{0.1}{2} [x_n^2 - y_n^2 + (x_n+0.1)^2 - (y_n + 0.1(x_n^2 - y_n^2))^2]$$

$$x_1 = 0.1 \quad y_1 = .91$$

$$x_2 = 0.2 \quad y_2 = .836800066195 \checkmark$$

Apply the method of variation of parameters

Homogeneous solution: $y_1 = e^{2x}, y_2 = e^{-2x}$

Particular solution: $y_p = Ax + B$

Substituting y_p into the differential equation

$(2A - 2(Ax + B)) - (2Ax + 2B) = 0$

$2A - 2Ax - 2B - 2Ax - 2B = 0$

$-4Ax - 2B + 2A = 0$

Modified
Euler's Method for $y'(t) = t^2 - y(t)^2$

$$y_{n+1} = y_n + h f\left(x_n + \frac{1}{2}h, y_n + \frac{1}{2}h f(x_n, y_n)\right)$$

$$h = 0.1$$

$$x_0 = 0 \quad y_0 = 1 \quad \text{~~1.0~~}$$

$$x_1 = 0.1$$

$$y_{n+1} = y_n + 0.1 \left[(x_n + \frac{1}{2}h)^2 - (y_n + \frac{1}{2}h(x_n^2 - y_n^2))^2 \right]$$

$$x_1 = 0.1$$

$$y_1 = .91$$

$$x_2 = 0.2$$

$$y_2 = .8367173880975 \quad \checkmark$$

$\frac{1}{2}x + \frac{1}{2}y = \frac{1}{2}$ (1) (1) + (2)
 $\frac{1}{2}x + \frac{1}{2}y = \frac{1}{2}$ (2)

$\frac{1}{2}x + \frac{1}{2}y = \frac{1}{2}$
 $\frac{1}{2}x + \frac{1}{2}y = \frac{1}{2}$

$1.0 = x$

$\left[\frac{1}{2}x + \frac{1}{2}y = \frac{1}{2} \right] 1.0 = y = 1.0$

$1.0 = y$

$1.0 = x$

The feasible region is bounded by the lines $x=0$, $y=0$, $x=1$, and $y=1$.

$1.0 = y$