Math 128A, Spring 2007

Homework 8 Solution

5.4.14 We approximate $f''(0.5)$ as 0.0800 and 0.0775 for $h = 0.1$ and $h = 0.2$, respectively. To understand the roundoff error, note that we know all of the data to within 0.00005. So,

$$f''(x) \approx \frac{f(x+h) - 2f(x) + f(x-h)}{h^2}$$
$$= \frac{\hat{f}(x+h) - 2\hat{f}(x) + \hat{f}(x-h)}{h^2} + \frac{\epsilon_1 - 2\epsilon_2 + \epsilon_3}{h^2}$$

where each $|\epsilon_i| < 0.00005$ and $\hat{f}(x)$ is the value in the table. The roundoff error term can be bounded

$$\left| \frac{\epsilon_1 - 2\epsilon_2 + \epsilon_3}{h^2} \right| < \frac{4 \times 0.00005}{h^2}$$

yeilding 0.0200 and 0.0050 for $h = 0.1$ and $h = 0.2$, respectively.

8.7.3 (a) If we let $Y_1 = Y$, $Y_2 = Y'$ and $Y_3 = Y''$, then the system

$$\begin{aligned} Y_1' &= Y_2 & Y_1(0) &= 1 \\ Y_2' &= Y_3 & Y_2(0) &= -1 \\ Y_3' + 4Y_3 + 5Y_2 + 2Y_1 &= 2x^2 + 10x + 8 & Y_3(0) &= 3 \end{aligned}$$

is equivalent to the original one.

(b) Letting $Y_1 = Y$ and $Y_2 = Y'$,

$$\begin{aligned} Y_1' &= Y_2 & Y_1(0) &= 3 \\ Y_2' + 4Y_2 + 13Y_1 &= 40\cos(x) & Y_2(0) &= 4 \end{aligned}$$

is equivalent to the original system.

8.7.4 If we let $X_1 = x$, $X_2 = x'$, $Y_1 = y$, $Y_2 = y'$, $Z_1 = z$, $Z_2 = z'$, and $R = (X_1^2 + Y_1^2 + Z_1^2)^{1/2}$, then

$$\begin{aligned} X_1' &= X_2 & X_2' &= -cX_1/R^3 \\ Y_1' &= Y_2 & Y_2' &= -cY_1/R^3 \\ Z_1' &= Z_2 & Z_2' &= -cZ_1/R^3 \end{aligned}$$

is equivalent to the original system.

Euler We get data

```
i x_i  y_i
0 0.00 1.0000
1 0.05 0.9500
2 0.10 0.9050
3 0.15 0.8645
4 0.20 0.8283
```