Math 128A, Spring 2007

Homework 5 Solution

5.2.8 (a) $I(f) = \int_a^b f(x)dx \approx \int_a^b f((a+b)/2)dx = (b-a)f((a+b)/2) = M_1(f)$.

(b) If we subdivide $[a, b]$ into n subintervals $a = x_0 < x_1 < \cdots < x_n = b$ with $x_{i+1} = x_i + h$ and apply M_1 to each subinterval, we get

$$\begin{aligned} I(f) &= \int_a^b f(x)dx \\ &= \int_{x_0}^{x_1} f(x)dx + \cdots + \int_{x_{n-1}}^{x_n} f(x)dx \\ &\approx (x_1 - x_0)f((x_0 + x_1)/2) + \cdots + (x_n - x_{n-1})f((x_{n-1} + x_n)/2) \\ &= hf(a + h/2) + \cdots + hf(a + (n - 1/2)h) \end{aligned}$$

(c) $M_1(1/(1+x)) = 2/3 \approx .6667$ and $M_2(1/(1+x)) = 2/5 + 2/7 \approx .6857$, while $I = \log 2 \approx .6931$.

5.2.10 (a) Following the derivation of Simpson's rule,

$$\begin{aligned} I(f) &= \int_a^b f(x)dx \\ &\approx \int_a^b P_4(x)dx \\ &= \int_a^b [f(x_0)L_0(x) + \cdots + f(x_4)L_4(x)]dx \\ &= C_0 f(x_0) + \cdots + C_4 f(x_4) \end{aligned}$$

where $C_i = \int_a^b L_i(x)dx$. These integrals are easily evaluated (but it is very tedious!), and you get Boole's rule.

(b) $B_4(1/(1+x)) = (2/45)*(1/4)*(7+32*4/5+13*2/3+32*4/7+7*1/2) \approx .7006$. This is just a little worse than $M_2(1/(1+x))$ from problem 8.