

Qualifying Exam Syllabus

Sarah Iveson

1:00 pm, July 19th, 2007

Committee: Mark Haiman, Michael Hutchings, Martin Olsson, Luca Trevisan (computer science).

1 Major Topic: Combinatorics (Algebra)

- Counting: ordinary generating functions, the Twelffold Way, inclusion-exclusion; exponential generating functions, Lagrange inversion, the Matrix-Tree theorem.
- Posets and lattices: semimodular and distributive lattices, incidence algebra, Möbius function and Möbius inversion, Eulerian posets, Euler characteristic, simplicial posets, Cohen-Macaulay posets.
- Symmetric functions: classical bases, Hall inner product, Schur functions, Robinson-Schensted-Knuth correspondence, Pieri formula, Jacobi-Trudi identity, hook length formula, Littlewood-Richardson rule.

Reference: R. Stanley, *Enumerative Combinatorics*, Vol. 1 (Chapters 1-4) and Vol. 2 (Chapters 5 & 7).

2 Major Topic: Algebraic Geometry (Algebra)

- Schemes: affine, irreducible, reduced, integral, Noetherian, finite type, base change.
- Morphisms: open/closed immersions, finite type, affine, finite, projective, separated, proper.
- Sheaves: quasi-coherent, coherent, $\mathcal{O}(n)$, invertible sheaves, locally free sheaves, ample, very ample.
- Divisors: Weil divisors, Cartier divisors, Picard group, morphisms to \mathbb{P}^n .
- Differentials: differentials, sheaves of differentials.
- Cohomology: Derived functor, cohomology of sheaves, cohomology of Noetherian affine schemes, Čech cohomology, cohomology of \mathbb{P}^n .
- Curves: Riemann-Roch theorem, Hurwitz's theorem, embeddings in projective space, the canonical embedding.

Reference: R. Hartshorne, *Algebraic Geometry*, Chapter II §1-8, Chapter III §1-5, Chapter IV §1-3, 5.

3 Minor Topic: Algebraic Topology (Geometry/Topology)

- Fundamental group: paths, homotopy equivalence, π_1 , Van Kampen's theorem, covering spaces, lifting properties.
- Homology: singular homology, relative homology, excision, cellular homology, Mayer-Vietoris, Euler characteristic, Borsuk-Ulam theorem, Lefschetz fixed point theorem.
- Cohomology: Universal coefficient theorem, cup product, orientations, Poincaré duality.

Reference: A. Hatcher, *Algebraic Topology* Chapter 1 §1-3, Chapter 2 §1, 3, A, B, C, Chapter 3 §1-3, A.