

# MANY CHEERFUL FACTS

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## Breaking $\mathbb{Q}[x_1, x_2, x_3, x_4]$ into tetris pieces

a talk by Jonah Blasiak

12:10 - 1:00pm on Wednesday, November 15th, in room 1015.

I'll draw lots of tetris pieces (actually, partitions of 4) on the board to convince you that there is plenty of interesting combinatorics and representation theory living in  $\mathbb{Q}[x_1, \dots, x_n]$ . I'll go over some basics of the representation theory of  $S_n$ , and I'll describe how the polynomial ring, with  $S_n$  acting by permuting the variables, decomposes as a sum of irreducible representations. What does multiplication in the polynomial ring look like in terms of this irreducible decomposition? This is not well understood in general, but a partial answer involves the  $q$ -Kostka polynomials, a generalization of the Kostka numbers which express Schur polynomials as a sum of monomial symmetric functions.

*I am the very model of a modern Major General,  
I've information vegetable, animal, and mineral,  
I know the kings of England, and I quote the fights historical  
From Marathon to Waterloo, in order categorical;  
I'm very well acquainted, too, with matters mathematical,  
I understand equations, both the simple and quadratical,  
About binomial theorem I'm teeming with a lot o' news,  
With many cheerful facts about the square of the hypotenuse!*

- Gilbert & Sullivan  $P \circ P$

The website for Many Cheerful Facts is  
<http://www.math.berkeley.edu/~siveson/cheerful/>