Qualifying Examination Syllabus

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Committe: M. Olsson, B. Poonen, P. Teichner (Chair), M. Gaillard.

1. Algebraic Geometry (Algebra)

Reference: Hartshorne, Algebraic Geometry, chapters II, §1 − §8, III, §1 − §8, IV, §1 − §4.

(a). Basic scheme theory, sheaf theory: sheaves on a topological space, sheafification, definition of schemes, separated/proper morphisms, (quasi-)coherent sheaves, affine/projective morphisms, ample sheaves, divisors, Picard groups, sheaf of differentials, $\Omega_{\mathbb{P}^n}$, (no formal schemes)

(b). Cohomology of sheaves: derived functors, Serre's criterion for affineness, Čech cohomology, $H^q(\mathbb{P}^n, \mathcal{O}(d))$, Serre's vanishing theorem, Serre's criterion for ampleness, statement of Serre duality (without proof), $R^q f_*$,

(c). Curves: linear systems, Riemann-Roch for curves, Hurwitz's formula, Frobenius morphisms, geometry of elliptic curves: j-invariant, group law, dual isogeny

2. Algebraic Number Theory (Algebra)

References: Serre, Local Fields, chapters I, III, VII, VIII, XII, XIII; Cassels and Fröhlich, Algebraic Number Theory, chapters VI, VII, and VIII, §3.

(a). Basic properties of number fields: ring of integers, normalized valuations, Chinese Remainder theorem, weak approximation, basic Galois theory, trace and norm, different and discriminant, ramification, explicit factorization of prime ideals, product formula, statement of Dirichlet's Unit theorem (without proof), Minkowski's bound, basic properties of cyclotomic fields and quadratic fields, statement of Čebotarev's density theorem as in Cassels and Fröhlich, Algebraic Number Theory, VIII, §3.

(b). Local fields: Hensel's lemma, unramified and totally ramified extensions

(c). Class field theory

• group cohomology: restrictions, inflations, corestrictions, induced modules, Shapiro's lemma

• class field theory: statement of main theorems (without proof), local and global reciprocity maps, Brauer groups of local and global fields, cohomology of idele groups and idele class groups

3. Algebraic Topology (Geometry/topology)

References: Hatcher, Algebraic Topology, chapters I, II, III; Massey, Algebraic Topology, chapters I to V.

(a). Basic topological notions: CW complexes, covering spaces, adjunction between loop and suspension

(b). Homotopy theory: homotopy groups, exact sequence of homotopy groups associated to a Serre fibration, relation between the fundamental group and covering spaces, Van-Kampen theorem

(c). Homology theory: singular/cellular homology and cohomology groups, Mayer-Vietoris sequences, universal coefficient theorems for cohomology and homology, cup-products, statement of Lefschetz fixed point theorem, Poincaré duality.