

6.10.

for (c), the hint is far from enough. Here's a key lemma one wants to prove.

Lemma 1. *Let \mathcal{R} be a coherent sheaf on $U := X - Y$, and let \mathcal{R}' and \mathcal{R}'' be two coherent extensions of \mathcal{R} to X . Then $\gamma(\mathcal{R}') - \gamma(\mathcal{R}'')$ is in the image of $K(Y)$.*

To show the lemma one can use (II, Ex.1.20b), together with some reductions and Hartshorne's hint. With this lemma it should be easier to prove the exactness in the middle. Come to my office hour for a complete proof.

6.11.

(b). To show there exists locally free resolutions of length 1, one may want to use some homological algebra, especially the fact that a finitely generated module over a PID has projective dimension ≤ 1 .

6.12.

Since $K(X)$ is the Grothendieck group of $\mathcal{Coh}(X)$, condition (3) means that the map deg should factor through $K(X)$, by the universal property of Grothendieck groups. Then use the previous exercise, which asserts $K(X) = \text{Pic } X \times \mathbb{Z}$, to define appropriate maps $\text{Pic } X \rightarrow \mathbb{Z}$ and $\mathbb{Z} \rightarrow \mathbb{Z}$, to prove the existence of the degree map. Uniqueness follows from (ex.6.11c).