

## Math 55 Quiz 1

Jan 27, 2016

1. Determine whether  $\forall x \exists y (P(x) \wedge Q(y))$  and  $\forall y \exists x (P(x) \wedge Q(y))$  are logically equivalent. If so, prove it. If not, provide a counterexample. (5 points)

2. Let  $P(x)$  be the statement “ $x$  is prime” and  $Q(x)$  be the statement “ $x$  is divisible by 5”. Determine the truth value of the following statements if the domain of each variable consists of all integers. Note that 1 is not prime. (1 point each)

- $\exists x P(x) \wedge Q(x)$

- $\forall x (Q(x) \leftrightarrow Q(x + 5))$

- $\forall x (P(x) \rightarrow \neg P(x - 1))$

- $\forall x (P(x) \vee Q(x))$

- $\exists x \exists y (P(x) \vee P(y)) \wedge (\neg P(x) \vee \neg P(y))$

**Problem of the Week:** Professor Srivastava announces in class that the first midterm will take place in a week and is expected to be very difficult. A student raises his hand.

“I have a yes or no question about the midterm, but it’s very important to me that you answer truthfully.”

“I promise to answer truthfully,” replies Professor Srivastava.

The student then asks a single, very clever question such that the only possible way for Professor Srivastava to answer truthfully requires for him to cancel the midterm. What was the question the student asked?

**Rules:** This problem is for your own personal enjoyment. You can take this paper home with you and work on it over the weekend. You can work with whoever else you want to work with, just let me know who you worked with. If you have an answer, let me know after class (either verbally or in writing) or by email. Whether or not you solve this problem has nothing to do with your grade in the class. The person, or people, who solved the most weekly problems will get a fun prize at the end of the semester. Enjoy!