Math 55 Quiz 10

April 20, 2016

1. Find the probability that when 8 dice are rolled, all 6 numbers come up.

2. Are the following relations on the set of ordered pairs of positive integers equivalence relations? Prove your answer. If so, describe the equivalence classes.

   - $\{\ ((a,b),(c,d))\ |\ \ a+d=b+c\}$

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3. Give an example of a relation that is reflexive and symmetric but not transitive.

Problem of the Week: Steve is piling $m \geq 1$ indistinguishable stones on the squares of an $n \times n$ grid. Each square can have an arbitrarily high pile of stones. After he finished piling his stones in some manner, he can then perform stone moves, defined as follows. Consider any four grid squares, which are corners of a rectangle, i.e. in positions $(i,k), (i,l), (j,k), (j,l)$ for some $1 \leq i,j,k,l \leq n$, such that $i < j$ and $k < l$. A stone move consists of either removing one stone from each of (i,k) and (j,l) and moving them to (i,l) and (j,k) respectively,j or removing one stone from each of (i,l) and (j,k) and moving them to (i,k) and (j,l) respectively.

Two ways of piling the stones are equivalent if they can be obtained from one another by a sequence of stone moves.

How many different non-equivalent ways can Steve pile the stones on the grid?

(source: USAMO)

Rules: This problem is for your own personal enjoyment. You can take this paper home with you and work on it over the weekend. You can work with whoever else you want to work with, just let me know who you worked with. If you have an answer, let me know after class (either verbally or in writing) or by email. Whether or not you solve this problem has nothing to do with your grade in the class. The person, or people, who solved the most weekly problems will get a fun prize at the end of the semester. Enjoy!