Math 55, Probability Worksheet #2
March 19, 2014

1. What is the probability of each of these events when we randomly select a permutation of the numbers one through four:

   - 1 precedes 4

   - 1 precedes 4 and 1 precedes 3

   - 1 precedes 4, 1 precedes 3, and 1 precedes 2

   - 1 precedes 4 and 2 precedes 3

2. What is the smallest number of people you can choose at random to guarantee that the probability that one of them has a birthday today is at least 1/2?

3. Prove that if E and F are events then $p(E \cap F) \geq p(E) + p(F) - 1$.

4. Prove that if E and F are independent events then $\overline{E}$ and F are also independent events.

5. Consider a family with n children. Let E be the event that the family has at least one child of each gender and let F be the event that the family as at most one boy. For which n (if any) are E and F independent? Assume that birth order matters, so the sets {boy, girl} and {girl, boy} are different outcomes.

6. Suppose that in the world of Harry Potter, 8% of people have magical powers, 96% of magical people have parents who are magical, and 9% of nonmagical people are squibs (i.e. have parents who are magical). What is the probability that a person selected randomly from Harry Potter's world with magical parents is magical himself?

7. Prove that if $E_1, E_2, \ldots, E_n$ are events from a finite sample space then

$$p(E_1 \cup E_2 \cup \cdots \cup E_n) \leq p(E_1) + p(E_2) + \cdots + p(E_n)$$

8. Suppose that 1 in every $10,000$ children are math prodigies. Say there is a test that can be given to babies at age 3 months that will determine if the baby will grow up to be a math prodigy or not (there is no such test of course, nor is there a well-defined notion of "prodigy", but let us assume for the sake of this problem). Say that 99% of math prodigies passed the test as babies and only 0.001% of people who are not math prodigies passed the test.

    - What is the probability that someone who passes the test grows up to be a math prodigy?

    - What is the probability that someone who doesn't pass the test doesn't grow up to be a math prodigy?