

**Math 104, Fall 07**  
**Homework#:6** Sequences

1. Give an example of a metric space which is not complete, i.e., in which there exists a Cauchy sequence which is not converge.
2. Let  $(X,d)$  be a compact metric space. Show that  $X$  is complete.
3. Let  $X$  be a complete metric space. Show that every closed subset  $F \subset X$  is also complete, i.e., show that if  $(x_n) \subset F$  is a Cauchy sequence then it has a limit  $L$  in  $F$ .
4. Consider the sequence  $x_{n+1} = \sqrt{2 + x_n}$  with  $x_1 = 1$ . Show that this sequence converge and compute its limit.
5. Show that a convergent sequence in a metric space is bounded.

**Good luck!!**