Level Set Methods
and
Fast Marching Methods

Evolving Interfaces in Computational Geometry, Fluid Mechanics, Computer Vision, and Materials Science

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Introduction

Propagating interfaces occur in a wide variety of settings, and include ocean waves, burning flames, and material boundaries. Less obvious boundaries are equally important and include shapes against backgrounds, handwritten characters, and iso-intensity contours in images. Furthermore, there are applications not commonly thought of as moving interface problems, including optimal path planning and construction of shortest geodesic paths on surfaces, which can be recast as front propagation problems with significant advantages.

The goal of this book is both to unify these ideas and to design a general framework for modeling the evolution of boundaries. The aim is to provide computational techniques for tracking moving interfaces and to give some hint of the flavor and breadth of applications. The work includes examples from physics, chemistry, fluid mechanics, combustion, image processing, materials sciences, fabrication of microelectronic components, computer vision, control theory, seismology, computer-aided-design, and a collection of other areas. The intended audience includes mathematicians, applied scientists, practicing engineers, computer graphics artists, and anyone interested in the evolution of boundaries and interfaces.

Our perspective comes from a large and rapidly growing body of work which relies on a partial differential equations approach for understanding, analyzing, and computing interface motion. At the core lay two computational techniques: "Fast Marching Methods" and "Level Set Methods". Both exploit a fundamental shift in how one views moving boundaries. They rethink the Lagrangian geometric perspective and replace it with an Eulerian, partial differential equation. Fast Marching Methods result from a boundary value problem for the evolving interface, while Level Set Methods result from an associated initial value problem.

In both cases, several advantages stem from this view of propagating interfaces:

- First, from a theoretical/mathematical point of view, some complexities of front motion are illuminated, in particular, the role of singularities, weak solutions, shock formation, entropy conditions, and topological change in the evolving interface.
- Second, from a numerical perspective, natural and accurate ways of computing delicate quantities emerge, including the ability to build high order advection schemes, compute local curvature in two and three dimensions, track sharp corners and cusps, and handle subtle topological changes of merger and breakage.
- Third, from an implementation point of view, since the approaches are based on underlying partial differential equations, robust schemes result from numerical parameters set at the beginning of the computation. The error is thus controlled by

  (i) the order of the numerical method,

  (ii) the grid spacing Δh,

  (iii) in the case of Level Set Methods, the time step Δt; no such requirement exists for Fast Marching Methods.

- Fourth, computational adaptivity is the key to these techniques. In the case of Level Set Methods, the most efficient and preferred approach is the "Narrow Band Level Set Method", which focuses computational labor around the evolving boundary. In the case of Fast Marching Methods, use of standard sorting techniques yields extraordinarily fast and optimally efficient algorithms. In both cases, a clear path to parallelism is available.

This book surveys what is intended to be an illustrative subset of past and current applications of these techniques. We do not assume that the reader is familiar with all of the details required to develop these schemes; the aim is to include the necessary theory and details to provide implementation guidelines.

The first edition of this book was entitled Level Set Methods. The augmented title Level Set Methods and Fast Marching Methods of this new edition embraces the large landscape shared by these two techniques in framing, illuminating, and solving problems with evolving boundaries.

Outline

This book is divided into four parts. Part I focuses on the formulation of the boundary value and initial value partial differential equations which comprise our two views of interface motion. Part II introduces the theory and numerics underlying Fast Marching Methods and Level Set Methods. Part III introduces the adaptive issues required to construct efficient schemes and variations on the fundamental techniques. Finally, Part IV surveys some application areas.

In **Part I**, Chapter 1 begins with the underlying boundary value and initial value partial differential equations perspective on moving interfaces, and discusses the theoretical and computational advantages of these approaches. It ends with a preview of the rest of the book and provides an outline of the interconnection of the techniques, the relevant theory, numerics, and application strategies. This "look ahead" is meant to provide a structure for the remainder of the book, directing the interested reader to various components of the methodologies.

Part II begins in Chapter 2 with a general statement of the problem of a moving interface and discusses the mathematical theory of curve/surface motion, including the growth/decay of total variation, singularity development, entropy conditions, weak solutions, and shocks in the dynamics of moving fronts. This material has been developed in a collection of papers that are referred to in the text. The viscosity theory of Hamilton-Jacobi equations, which buttresses both computational techniques, is briefly surveyed in Chapter 3.

Chapters 4, 5, and 6 present numerical results which lead up to the Fast Marching and Level Set techniques. Chapter 4 begins with an overview of traditional methods for tracking interfaces, including string methods and cell methods, and makes a first attempt at solving a partial differential equation for front propagation The failure of this first attempt stems from the relationship between front propagation and hyperbolic conservation laws and is the subject of Chapter 5. Chapter 6 then provides a detailed description of straightforward (though inefficient) algorithms for solving the initial value and boundary value problems.

Part III provides complete details on state-of-the-art Fast Marching and Level Set algorithms. It begins in Chapter 7 with a discussion of computational adaptivity. After surveying work on parallel and adaptive mesh approaches, the chapter focuses on the Narrow Band Level Set Method. This is the most efficient and accurate way to implement level set methods. Next, in Chapter 8, Fast Marching Methods are introduced, which are the optimal way to solve Hamilton-Jacobi equations which arise from certain interface motion problems. The techniques require a detailed discussion of causality in upwind schemes and optimal heap sort algorithms. Higher accuracy versions of both Narrow Band and Fast Marching Methods are supplied.

Next, in Chapters 9 and 10, the entire framework is moved to a triangulated unstructured mesh setting. Schemes for the Level Set Method are given, including monotone schemes, positive schemes, Petrov-Galerkin schemes, as well as explicit and implicit schemes with discontinuity capturing. In the case of Fast Marching Methods, upwind causality schemes for both acute and non-acute triangulations are introduced. These two sets of schemes provide versatile techniques for interface propagation problems on manifolds and in irregular domains.

Chapter 11 explains how to build general level set methods in many physical problems. It examines how to build appropriate and natural methods for moving the neighboring level sets, which is required in order to implement level set techniques. Detailed techniques for generating smooth level set flows which avoids all re-initialization are given, as are techniques for obtaining sub-grid accuracy. In Chapter 12, the numerical accuracy and robustness tests are measured, including scheme convergence rates, tests of triangulated techniques, examination of mass conservation and accuracy. Finally, in Chapter 13, the underlying philosophy of Narrow Band and Fast Marching Methods applications is discussed.

Part IV focuses on applications of both the Narrow Band Level Set Method and Fast Marching Methods to a collection of problems. Here, the intent is to show the breadth of current applications and to serve as a guidepost for further research. Chapter 14 begins with some pure geometry problems, including curve/surface shrinkage, the existence of self-similar surfaces, flows under more complex metrics, sintering and second derivative of curvature flows, triple points, multiple interfaces, and constraint-based flows. Chapter 15 extends this work and shows how these techniques can be used in grid generation, giving many examples of body-fitted logical rectangular grids around complex bodies in two and three dimensions. Chapter 16 moves to image processing and views images as collections of iso-intensity contours; by constructing a suitable speed law, these contours can be allowed to propagate in a way that both removes noise and enhances desired regions.

Chapter 17 focuses on aspects of computer vision. It begins with

Part I

Equations of Motion for Moving Interfaces

Part I presents the underlying partial differential equations perspective on moving interfaces. One view leads to a boundary value partial differential equation for the evolving front, the other leads to a time-dependent initial value problem. The goal is to lay out clearly the two views and discuss the theoretical and computational advantage of these approaches.

1

Formulation of Interface Propagation

Outline: *We formulate the boundary value and initial value partial differential equations which describe interface motion. These will eventually lead to the Fast Marching Method and the Narrow Band Level Set Method; for now, however, we focus on the theoretical and computational advantages that come from these perspectives.*

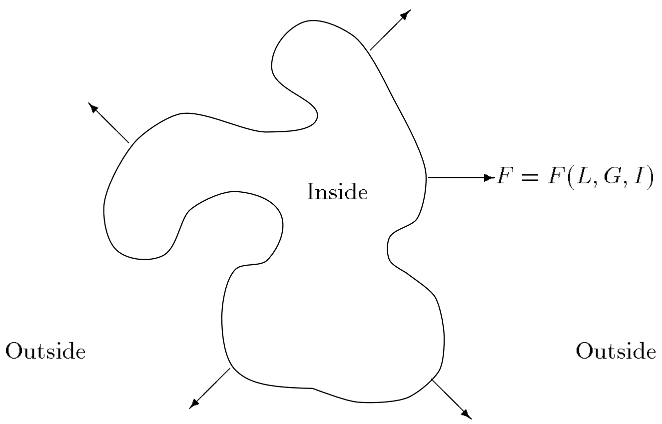


Fig. 1.1. Curve propagating with speed F in normal direction.

Consider a boundary, either a curve in two dimensions or a surface in three dimensions, separating one region from another. Imagine that this curve/surface moves in a direction normal to itself (where the normal direction is oriented with respect to an inside and an outside) with a known speed function F. The goal is to track the motion of this interface as it evolves. We are concerned only with the motion of the interface in its normal direction; throughout, we shall ignore motions of the interface in its tangential directions.

The speed function F, which may depend on many factors, can be written as:

$$F = F(L, G, I), \tag{1.1}$$

where

- *L= Local properties* are those determined by local geometric information, such as curvature and normal direction.
- *G= Global properties of the front* are those that depend on the shape and position of the front. For example, the speed might depend on integrals along the front and/or associated differential equations. As a particular case, if the interface is a source of heat that affects diffusion on either side of the interface, and a jump in the diffusion in turn influences the motion of the interface, then this would be characterized as global property.
- *I= Independent properties* are those that are independent of the shape of the front, such as an underlying fluid velocity that passively transports the front.

Much of the challenge in interface problems comes from producing an adequate model for the speed function F; this is a separate issue independent of the goal of an accurate scheme for advancing the interface based on the model for F. In this chapter, it is assumed that the speed function F is known. The goal of Part IV is to formulate good models for F for a collection of applications.

Given F and the position of an interface, the objective is to track the evolution of the interface. Our first task is to formulate this evolution problem in an *Eulerian* framework, that is, one in which the underlying coordinate system remains fixed.

1.1 A boundary value formulation

Assume for the moment that $F > 0$, hence the front always moves "outward." One way to characterize the position of this expanding front is to compute the arrival time $T(x, y)$ of the front as it crosses each point (x, y), as shown in Figure 1.2.

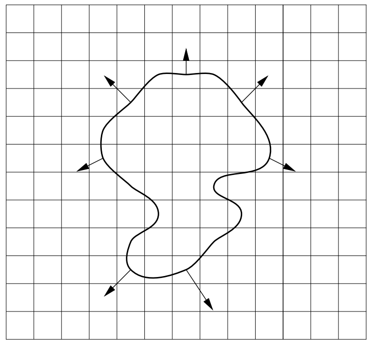


Fig. 1.2. Calculation of crossing time at (x, y) for expanding front $F > 0$.

The equation for this arrival function $T(x, y)$ is easily derived. In one dimension, using the fact that *distance* = *rate* $*$ *time* (see Figure 1.3), we have that

$$1 = F \frac{dT}{dx}.$$

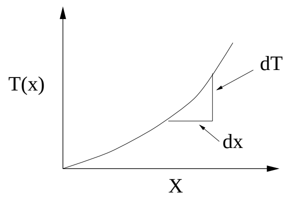


Fig. 1.3. Setup for boundary value formulation.

In multiple dimensions, ∇T is orthogonal to the level sets of T, and, similar to the one-dimensional case, its magnitude is inversely proportional to the speed. Hence

$$|\nabla T|F = 1, \qquad T = 0 \text{ on } \Gamma, \tag{1.2}$$

where Γ is the initial location of the interface.

Thus, the front motion is characterized as the solution to a boundary value problem. If the speed F depends only on position, then the equation reduces to what is known as the "Eikonal" equation. As an example, the arrival surface $T(x, y)$ for a circular front expanding with unit speed $F = 1$ is shown in Figure 1.4.

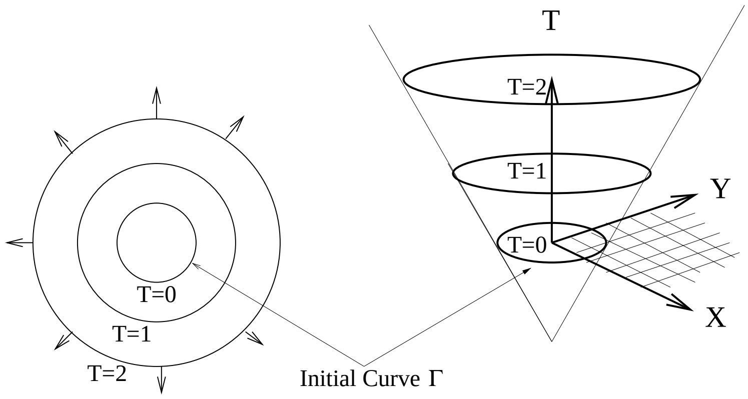


Fig. 1.4. Transformation of front motion into boundary value problem.

1.2 An initial value formulation

Conversely, suppose now that the front moves with a speed F that is neither strictly positive nor negative. Then we must account for the fact that the front can move forward and backward, and hence can pass over a point (x, y) several times. Thus, the crossing time $T(x, y)$ is not a single-valued function. Our way of taking care of this is to embed the initial position of the front as the zero level set of a higher-dimensional function ϕ. We can then link the evolution of this function ϕ to the propagation of the front itself through a time-dependent initial value problem. At any time, the front is given by the zero level set of the time-dependent level set function ϕ (see Figure 1.5).

In order to derive an equation of the motion for this level set function ϕ and match the zero level set of ϕ with the evolving front, we first require that the level set value of a particle on the front with path $x(t)$ must always be zero, and hence

$$\phi(x(t), t) = 0. \tag{1.3}$$

By the chain rule,

$$\phi_t + \nabla\phi(x(t), t) \cdot x'(t) = 0. \tag{1.4}$$

Since F supplies the speed in the outward normal direction, then $x'(t) \cdot n = F$, where $n = \nabla\phi/|\nabla\phi|$. This yields an evolution equation for ϕ, namely

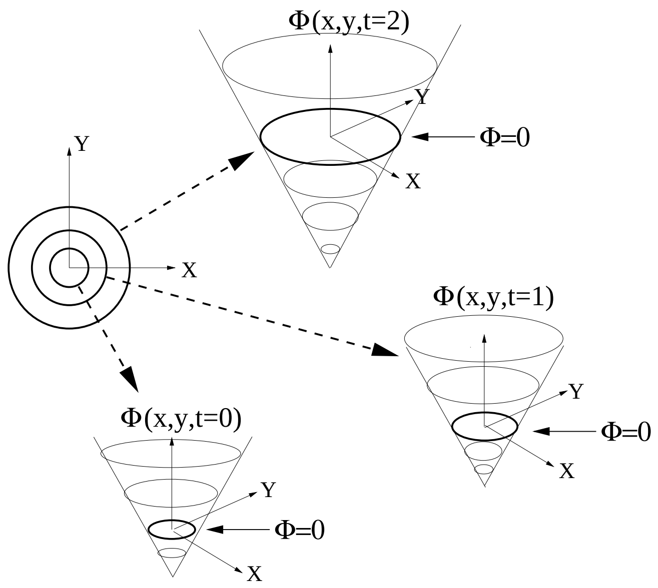


Fig. 1.5. Transformation of front motion into initial value problem.

$$\phi_t + F|\nabla\phi| = 0, \tag{1.5}$$

$$\text{given} \quad \phi(x, t = 0).$$

This is the level set equation given by Osher and Sethian [187]. For certain forms of the speed function F, one obtains a standard Hamilton–Jacobi equation. Equation 1.5 describes the time evolution of the level set function ϕ in such a way that the zero level set of this evolving function is always identified with the propagating interface; see Figure 1.5.

Thus, we can summarize our two perspectives. Let Γ be a curve in the plane propagating in a direction normal to itself with speed F such that $\Gamma(t)$ gives the position of the front at time t. Then, we wish to solve

Boundary Value Formulation	**Initial Value Formulation**
$\lvert\nabla T\rvert F = 1$	$\phi_t + F\lvert\nabla\phi\rvert = 0$
Front= $\Gamma(t) = \{(x,y)\lvert T(x,y) = t\}$	Front=$\Gamma(t) = \{(x,y)\lvert\phi(x,y,t) = 0\}$
Requires $F > 0$	Applies for arbitrary F

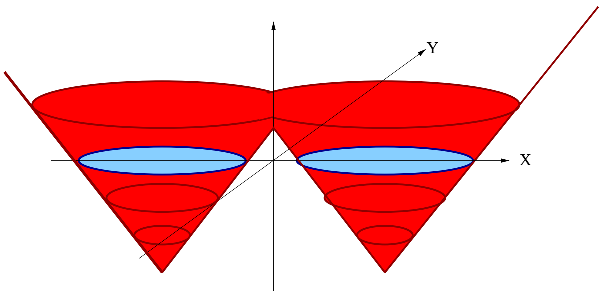
(1.6)

1.3 Advantages of these perspectives

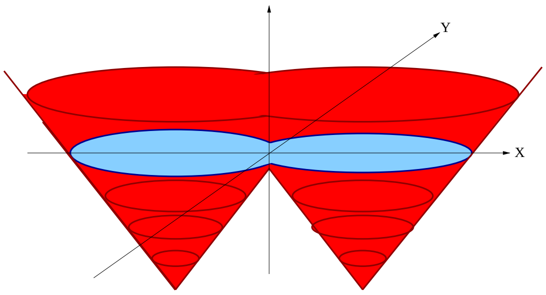
There are certain advantages associated with these two perspectives on propagating interfaces.

- Both are unchanged in higher dimensions, that is, for hypersurfaces propagating in three dimensions and higher.

- Topological changes in the evolving front Γ are handled naturally. The position of the front at time t is given either by the zero level set $\phi(x,y,t)=0$ of the evolving function ϕ or by the level set $T(x,y)=t$ of the boundary value solution. This set need not be a single curve, and it can break and merge as t advances. In both cases, the key fact is that the boundary value solution $T(x,y)$ and the level set function ϕ remain single-valued (see Figure 1.6).



The level set surface ϕ (dark gray):
Two separate initial fronts (in light gray).



Later in time: the interface topology has changed, yielding a single curve as the zero level set.

Fig. 1.6. Topological change.

- Both rely on viscosity solutions of the associated partial differential equations in order to guarantee that the unique, entropy-satisfying weak solution is be obtained.

- Both are accurately approximated by computational schemes which exploit techniques borrowed from the numerical solutions of hyperbolic conservation laws. For example, schemes may be developed by using a discrete grid in x-y domain and substituting finite difference approximations[1] for the spatial and temporal derivatives. As illustration, using a uniform mesh of spacing h, with grid nodes (i, j), and employing the standard notation that ϕ_{ij}^n is the approximation to the solution $\phi(ih, jh, n\Delta t)$, where Δt is the time step, one might write
$$\frac{\phi_{ij}^{n+1} - \phi_{ij}^n}{\Delta t} + F|\nabla_{ij}\phi_{ij}^n| = 0. \tag{1.7}$$
Here, a forward difference scheme in time has been used, and $|\nabla_{ij}\phi_{ij}^n|$ represents some appropriate finite difference operator for the spatial derivative. Thus, an explicit finite difference approach is possible. The construction of correct entropy-satisfying approximations to the difference operator is the subject of Part II; for now, the important fact is that one has an explicit error control on the basis of the initial spatial discretization and the order of the numerical scheme.

- Intrinsic geometric properties of the front are easily determined in both formulations. For example, at any point of the front, the normal vector is given by
$$\vec{n} = \frac{\nabla\phi}{|\nabla\phi|} \quad \text{or} \quad \vec{n} = \frac{\nabla T}{|\nabla T|}, \tag{1.8}$$
and the curvature of the front at any point is easily obtained from the divergence of the unit normal vector to the front, i.e.,
$$\kappa = \left\{ \begin{array}{l} \nabla \cdot \frac{\nabla\phi}{|\nabla\phi|} = \frac{\phi_{xx}\phi_y^2 - 2\phi_x\phi_y\phi_{xy} + \phi_{yy}\phi_x^2}{(\phi_x^2+\phi_y^2)^{3/2}} \\ \nabla \cdot \frac{\nabla T}{|\nabla T|} = \frac{T_{xx}T_y^2 - 2T_xT_yT_{xy} + T_{yy}T_x^2}{(T_x^2+T_y^2)^{3/2}} \end{array} \right\}. \tag{1.9}$$

- Both methods are made efficient through the use of adaptive computational strategies, which lead to Narrow Band Level Set Methods and Fast Marching Methods.

[1] Finite difference approximations will be discussed in detail in Chapter 5.

At the same time, there are significant differences between the two approaches.

- The most obvious difference is that the initial value level set formulation allows for both positive and negative speed functions F; the front may move forward and backward as it evolves. The boundary value perspective is restricted to fronts that always move in the same direction, because it requires a single crossing time T at each grid point, and hence a point cannot be revisited. Thus, models involving more complex speed functions F, such as those including curvature, are most naturally framed as initial value level set problems.
- Conversely, positive speed functions F which depend on position and vary widely from point to point are best framed as boundary value problems and approximated through the use of Fast Marching Methods. This is because
  - (i) The boundary value formulation requires no time step, and hence its approximation is not subject to CFL conditions, unlike Level Set Methods.
  - (ii) Through the use of heap sort algorithms, Fast Marching Methods can be made extremely computationally efficient, far eclipsing Level Set Methods.

1.4 A general framework

We can be slightly informal and describe both formulations with the general partial differential equation

$$\alpha u_t + H(Du, x) = 0. \tag{1.10}$$

Here, Du represents the partials of u in each variable, for example, u_x and u_y. In the case of the Eikonal equation, $\alpha = 0$, and the function H reduces to $H = F|\nabla u| - 1$.

One of the main subtleties that arises in solving this equation is that the solution need not be differentiable, even with arbitrarily smooth boundary data. This non-differentiability is intimately connected to the notion of appropriate weak solutions. Our goal will be to construct numerical techniques which naturally account for this non-differentiability in the construction of accurate and efficient approximation schemes and admit physically correct non-smooth solutions.

1.5 A look ahead/A look back

It is worthwhile to stop and explain how these techniques were developed and what lies ahead. The first step in the development of these ideas started with the analysis of corners and singularities in propagating interfaces. In [222, 225], the role of curvature as a regularizing or smoothing term was investigated, and it was shown that this regularizing role connects to the notion of entropy conditions and shocks in hyperbolic conservation laws in gas dynamics. This is the subject of Chapter 2. A more formal view comes from considering viscosity solutions of Hamilton-Jacobi equations, which is the subject of Chapter 3.

The second step in the development of accurate and efficient numerical techniques for interface evolution comes from the realization that the schemes from computational fluid mechanics, specifically designed for approximating the solution to hyperbolic conservation laws, can be used to solve the equations of front propagation. This was the view developed in [226], and is at the core of modern interface methods:

> "Most algorithms place marker particles along the front and advance the position of the particles in accordance with a set of finite difference approximations to the equations of motion. Such schemes usually go unstable and blow up as the curvature builds around a cusp, since small errors in the position produce large errors in the determination of the curvature. One alternative is to consider the reformulation equations of motion as a conservation law with viscosity and solve these equations with the techniques developed for gas dynamics. These techniques, based on high-order upwind formulations, are particularly attractive, since they are highly stable, accurate and preserve monotonicity. We have made some preliminary tests of such schemes applied to our problem of propagating fronts in crystals and flames, with extremely encouraging results..."

To execute this strategy, we need schemes from hyperbolic conservation laws; this is the subject of Chapters 4 and 5.

The combination of these three subjects then leads to the two numerical schemes given in Chapter 6: the Level Set Method ([187]) for the initial value problem, and an iterative method for the boundary value problem. They are made efficient in Chapters 8 and 9 through adaptivity, leading to Narrow Band Level Set Methods, see [2], and Fast Marching Methods, see [233]. Finally, after a series of extensions of the basic techniques are developed in Part III, many applications are described in Part IV.

The interconnectedness of past work on methods for interface propagation and the set of applications to be discussed are shown in Figure 1.7. There are many other contributors to the evolution of these ideas; the chart is meant to give perspective on how the theory, algorithms, and applications have evolved. We urge the reader to consult the bibliography to get a more complete sense of the literature and the range of work underway.

1.6 A larger perspective

Fast Marching Methods and Level Set Methods offer powerful techniques for tracking moving interfaces. This book (like its previous edition) aims to demonstrate how these techniques are applied across a wide spectrum of applications. However, there are many other ways to compute solutions to these problems besides the techniques offered here. Marker particle techniques, Volume–of–fluid simulations, Fourier techniques, and phase field models all offer valuable approaches. For each application area given, there is a substantial literature which describes other approaches. At the same time, new techniques and algorithms are always under development, and one of the surest ways to render a body of work obsolete is to pronounce that it can't be improved upon. With that in mind, our goal here is to capture the flavor, intuitive feel, and details of Fast Marching and Level Set Methods.

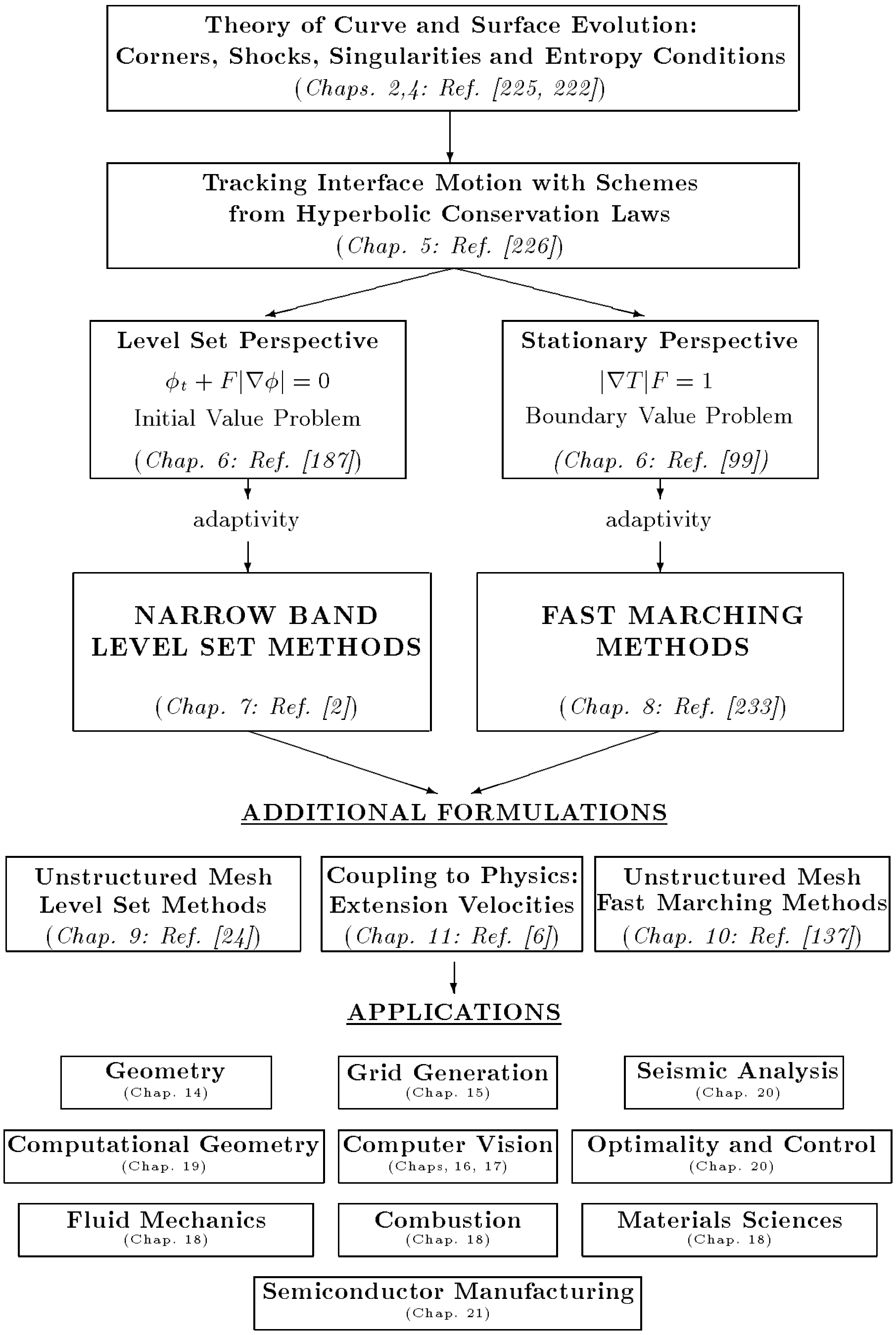


Fig. 1.7. Evolution of theory and algorithms for interface propagations.

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