Department of Mathematics
University of California, Berkeley

Mathematics 261A
Lie Groups

Vera Serganova, Fall 2002

My **office hours** are 4:30-5:30 on Mondays and 11-1 on Wednesdays, in 709 Evans Hall, email at `serganov@math`. You are welcome to ask questions by email.

The **texts** for this course are **Fulton, Harris, Representation Theory** and **Humphreys, Introduction to Lie Algebras and Representation Theory**.

To understand this course you need basic knowledge of Algebra and Differential Geometry. In other words you have to know what is a group and what is a differentiable manifold.

Each Friday I will post on my web page a problem assignment (3-5 problems) on the material of the week lectures (http://math.berkeley.edu/ serganov). The homework will be collected the next Friday.

The **grade** will be computed according to the following proportions: 50% for your homework and 50% for the take home final. But if you solve all problems in your final (there will be hard ones in it) you get A for the course.

Course outline

- Lie Groups. Definitions and examples. Fundamental group of a Lie group. Covering
- Lie Algebras and Exponential Map. Relation between subgroups and subalgebras. Campbell-Hausdorff formula.
- Structure Theory of Lie Algebras. Solvable and Nilpotent Algebras. Engel's and Lie's theorems. Semisimple Lie Algebras. The Killing Form and Cartan's Criterion. Jordan decomposition
- Ado's and Levi's theorems
- Representations of Classical Lie Groups. Representations of sl_2 and sl_3. Representations of Gl_n and Weyl Duality. Spin Representations of Orthogonal Groups
- Universal Enveloping Algebras. PBW Theorem
- Classification of Complex Semisimple Lie Algebras. Root Systems, Dynkin Diagrams and Weyl Groups. Exceptional Lie Algebras
- Representations of Semisimple Lie Algebras. Casimir Operator and Complete Reducibility. Weyl Character Formula
- Compact Lie Groups and Their Representations. Peter-Weyl Theorem
- Compact Homogeneous Spaces and Bruhat Decomposition