

**Sample final**  
**Math 1B**

**Whereabouts:** Final exam will take place on Friday, May 16.  
at 8 : 00 – 11 : 00 AM in 100 Haas pavilion.

The final will cover all material of the course with more stress (about 50 %) on chapters 8 and 15. Calculators and books may not be used during the test. All formulas which are hard to memorize will be given. The final score is 40% of your grade for the class.

**Sample test #1.**

1. Evaluate the integral

$$\int \frac{\cos x}{e^{\sin x} + 1} dx.$$

2. Evaluate the integral if it converges or show that it is divergent

$$\int_0^1 \frac{dx}{x\sqrt{x+1}}.$$

3. Determine whether the sequence is convergent or divergent. If it is convergent find the limit:

$$a_n = \left(1 + \frac{1}{n^2}\right)^n.$$

Hint: check the sequence  $\ln a_n$ .

4. A spring has a natural length of  $1m$ . A force of  $3N$  is required to keep it stretched to a length  $1.2m$ . How much work is required to stretch the spring from  $1.2m$  to  $1.5m$ .

5. Determine whether the series is convergent or divergent

$$\sum_{n=1}^{\infty} \frac{(-1)^n n^{1/3}}{n^2 + 1}.$$

6. Find the radius and the interval of convergence for the power series:

$$\sum_{n=0}^{\infty} \frac{x^n}{3^n n^2}.$$

7. Solve the differential equation

$$yy' = x\sqrt{1+x^2}\sqrt{1+y^2}.$$

8. Solve the differential equation  $y'' - 2y = e^{2x}$ .

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*Date:* May 3, 2008.

9. Solve the initial value problem using power series

$$y'' - xy = 0, y(0) = 1, y'(0) = 0.$$

10. A spring with mass of  $3kg$ , is stretched  $0.3m$  beyond its original length and then released with zero velocity. Assume that the damping constant is 11 and a force of  $3N$  is required to keep the spring stretched  $0.3m$  beyond its original length. Find the position of the mass as a function of time.

**Sample final #2.**

1. Find the integral or show that it is divergent

$$\int_0^{\infty} \frac{dx}{(x+1)^2(x+2)}.$$

2. Determine whether the sequence  $a_n = \frac{\ln^2 n}{n}$  is convergent or divergent. If it is convergent, find the limit.

3. Determine whether the series is divergent or convergent

$$\sum_{n=1}^{\infty} \frac{n^2 - n}{n^4 + n}.$$

4. Let  $f(x) = x^2 e^{x^3}$ . Find  $f^{(100)}(0)$ .

5. A tank contains  $1000L$  of pure water. Brine that contains  $0.05kg$  of salt per liter of water enters the tank at a rate of  $10L/\text{min}$ . Brine that contains  $0.02kg$  of salt per liter of water enters the tank at a rate  $5L/\text{min}$ . The solution is kept thoroughly mixed and drains from the tank at a rate of  $15L/\text{min}$ . How much salt is in the tank after 1 hour?

6. Find the radius and interval of convergence of the power series

$$\sum_{n=0}^{\infty} \frac{(n!)^2}{(2n)!} x^n.$$

7. Solve the differential equation  $y'' - xy' - 2y = 0$  in power series.

8. Solve the initial value problem

$$y'' - 2y' - 3y = e^x \cos 4x, y(0) = y'(0) = 0.$$

9. Use method of variation of parameters to solve the differential equation

$$y'' - y = \frac{1}{\cosh x}.$$

10. Estimate the error of approximation  $\ln(1.2) \approx 0.2$  using Taylor's formula.

**Sample test # 3.**

1. Evaluate the integral

$$\int (8 + 2x - x^2)^{1/2} dx.$$

2. A circular vertical plate of radius  $2m$  is submerged in water so that exactly half of the plate is under water. Find the hydrostatic force against one side of the plate.

3. Determine if the series is conditionally convergent, absolutely convergent or divergent.

$$\sum_{n=1}^{\infty} (-1)^n \left( \frac{n+1}{n^2+1} \right)^{1/3}.$$

4. (a) Let

$$f(x) = \sum_{n=1}^{\infty} nx^n.$$

Find a closed formula for  $f(x)$ .

(b) Find the sum of the infinite series

$$\sum_{n=1}^{\infty} \frac{n}{3^n}.$$

5. Find the Taylor polynomial  $T_4(x)$  for the function  $f(x) = \sqrt{1+x^2}$  at  $a = 0$ .

6. Find the centroid of the plane region bounded by the parabola  $y = 1 - x^2$  and the line  $y = -3$ .

7. Express the repeating decimal  $1.727272\dots$  as a fraction.

8. A force of  $10N$  is needed to keep a string with  $1kg$  mass stretched  $0.1m$  beyond its natural length. Find the damping constant that would produce critical damping.

9. Solve the initial value problem

$$9y'' + y = 3x + e^{-x}, \quad y'(0) = 2, \quad y(0) = 1.$$

10. Solve the boundary value problem

$$y'' + 9y = \cos x, \quad y(0) = 1, \quad y(\pi) = 0.$$