PRACTICE MIDTERMS

Test 1.

1. Evaluate the following integrals:

$$(a) \int \sin 2x \sin x dx$$

$$(b) \int \frac{dt}{t^2+t^3}$$

$$(\boldsymbol{c}) \int \frac{x^{1/2}dx}{1+x^{1/4}}$$

$$(\boldsymbol{d}) \int \cos\sqrt{x}dx$$

2. Determine whether each improper integral is convergent or divergent. Evaluate the integrals which are convergent:

$$(a) \int_{-1}^{1} \frac{dx}{x^3}$$

$$(b) \int_0^\infty xe^{-x}dx$$

3. Determine how large do we have to choose n to evaluate

$$\int_{-2}^{-1} e^{1/x}dx$$

with an error less than 0.01 using the trapezoidal rule. Write formula for this approximation. Do not evaluate! Use the formula $|E_T| \le \frac{(b-a)^3K}{12n^2}$ if $|f''(x)| \le K$ for $a \le x \le b$.

4. Determine whether the integral

$$\int_{-\infty}^{\infty} e^{-x^2-2x}dx$$

is convergent or divergent. Justify your answer. Do not try to evaluate this integral!

Test 2.

1. Evaluate the following integrals:

$$(a) \int \sin^3 x \cos x dx$$

$$(b) \int \frac{x^3}{(x-1)^{13}}dx$$

$$(\boldsymbol{c})\int \frac{dx}{e^{2x}+3e^{x}+2}$$

$$(\boldsymbol{d})\int \left(4-x^{2}\right)^{1/2} dx$$

2. Let

$$I_n=\int_0^{\pi/2}\sin^n x dx.$$

Show that for $n>1$

$$I_n=\frac{n-1}{n}I_{n-2}.$$

3. Determine how large do we have to choose n to evaluate

$$\int_0^{10}\cos x^2 dx$$

with an error less than 10^{-5} using the midpoint rule. Use the formula $|E_M|\leq\frac{(b-a)^3K}{24n^2}$ if $|f''(x)|\leq K$ for $a\leq x\leq b$.

4. Determine whether the integral

$$\int_0^{\pi/2}\sec x dx$$

is convergent. Justify your answer.

Test 3. 1. Evaluate the following integrals:

$$(a)\int\frac{\ln x}{x(\ln x+1)}dx$$

$$(b)\int\frac{x^2}{(x+1)^{2008}}dx$$

$$(\boldsymbol{c})\int\frac{1-\tan x}{1+\tan x}dx$$

2. Evaluate the area under the curve $y=e^{2x}\sin x$, $0\leq x\leq\pi$.

3. Estimate the error of evaluating the integral

$$\int_0^1\cos x^2 dx$$

using the midpoint rule with $n=100$.

4. Determine whether the integral

$$\int_0^{\infty}\frac{dx}{x^2+4x+3}dx$$

is convergent. Justify your answer. If the integral is convergent, evaluate it.