

PROBLEM SET # 1
MATH 252

Due November 4.

1. Let R be the algebra of polynomial differential operators. In other words R is generated by x and $\frac{\partial}{\partial x}$ with relation

$$\frac{\partial}{\partial x}x - x\frac{\partial}{\partial x} = 1.$$

(The algebra R is called the Weyl algebra.) Let $M = \mathbb{C}[x]$ have a structure of R -module in the natural way. Show that $\text{End}_R(M) = \mathbb{C}$, M is an irreducible R -module and the natural map $R \rightarrow \text{End}_{\mathbb{C}}(M)$ is not surjective.

2. Let R be a subalgebra of upper triangular matrices in $\text{Mat}_n(\mathbb{C})$. Classify simple and indecomposable projective modules over R and evaluate $\text{Ext}_R^i(M, N)$ for all simple M and N .