

**PROBLEM SET # 6**  
**MATH 252**

Due October 14.

Use notations of lecture notes 5 and 6.

**1.** Let  $\lambda = (n - k, 1, \dots, 1)$  be the hook diagram with  $n$  boxes. Show that  $V_\lambda$  is isomorphic to  $\Lambda^k V$ , where  $V$  is  $n - 1$ -dimensional subrepresentation in the standard permutation representation of  $S_n$ . (Hint: Use Lemma 5.12 in Lecture notes 5)

**2.** Show that the  $GL_k$ -representation  $W_\lambda \otimes E$  decomposes into direct sum of  $W_\mu$  for all  $\mu \in \Gamma_{n+1, k}$  which can be obtained from  $\lambda$  by adding one box. (Hint: check when  $\rho_{c_\mu} \rho_{c_\lambda}(E^{\otimes n+1}) \neq 0$ .)