

PROBLEM SET # 5
MATH 252

Due October 7.

1. Classify irreducible representations of A_4 (even permutations) over \mathbb{R} and over \mathbb{C} . What is the splitting field for A_4 .
2. Let G be a finite group, r be the number of conjugacy classes in G and s be the number of conjugacy classes in G preserved by the involution $g \rightarrow g^{-1}$. Prove that the number of irreducible representations of G over \mathbb{R} is equal to $\frac{r+s}{2}$.
3. If λ is a Young tableau, then the conjugate tableau λ' is obtained from λ by symmetry about diagonal (rows and columns switch). Show that $V_{\lambda'}$ is isomorphic $V_{\lambda} \otimes \text{sgn}$, where sgn is one-dimensional sign representation. (Hint: you probably have to show that $\mathbb{Q}(S_n) a_{\lambda} b_{\lambda}$ and $\mathbb{Q}(S_n) b_{\lambda} a_{\lambda}$ are isomorphic).