**PROBLEM SET # 9**
**MATH 252**

Due April 14.

**1**. Finish the proof of Krull-Schmidt theorem (see lecture notes 9).

**2**. Let $M_\alpha$ be a set of simple modules over a ring $R$. Show that if $\prod M_\alpha$ is semisimple then there are finitely many non-isomorphic modules among $M_\alpha$. Show that converse is not always true.

**3**. Let $S$ be a subring of a ring $R$. Show that if $P$ is a projective $S$-module then the induced module $R \otimes_S P$ is a projective $R$-module. (Hint: use Frobenius reciprocity $\mathrm{Hom_R(R \otimes_S P, M) = Hom_S(P, M)}$.

**4**. An $R$-module $I$ is injective if for any injective homomorphism of $R$-modules $f : M \to N$ and for any homomorphism $\phi : M \to I$ there exists a homomorphism $\psi : N \to I$ such that $\phi = \psi \circ f$.

(a) Show that $I$ is injective if and only if for any embedding $I \subset L$ there exists a submodule $I' \subset L$ such that $L = I \oplus I'$.

(b) Show that $\mathbb{Q}$ is an injective module over $\mathbb{Z}$.

*Date*: April 10, 2011.