PROBLEM SET # 1
MATH 252

Due January 27.

1. List all up to isomorphism irreducible representation of $\mathbb{Z}$ over $\mathbb{C}$.

2. Show that the set of isomorphism classes of all one dimensional representations of a group G over an arbitrary field k form an abelian group with respect to the operation of tensor product.

3. Let S_n denote the symmetric group and let $\rho : G \to GL(V)$ be the permutation representation of S_n in k^n. Show that if the characteristic of k does not divide n, then ρ is a direct sum of two irreducible representations.

4. Let $G = H \times K$ be a direct product of two finite groups. Let $\rho : H \to GL(V)$ and $\sigma : K \to GL(W)$ be irreducible representations. Show that if k is algebraically closed then the exterior tensor product $\rho \boxtimes \sigma$ is irreducible. Give a counterexample if k is not algebraically closed.

Date: January 19, 2011.