

**TAKE HOME FINAL**  
**MATH 252**

Due May 11.

**1.** Let  $G$  be a finite group and  $\rho : G \rightarrow GL(V)$  be a finite-dimensional representation of  $G$  over a field of characteristic zero. Prove the identity

$$\sum_{k=0}^{\infty} \dim S^k(V)^G t^k = \frac{1}{|G|} \sum_{g \in G} \frac{1}{\det(1 - t\rho_g)}.$$

**2.** Let  $G$  be the group of symmetries of the  $n$ -dimensional cube. Classify irreducible representations of  $G$  over  $\mathbb{C}$ .

**3.** Let  $E$  denote an  $n$ -dimensional vector space over  $\mathbb{C}$ . Decompose  $\Lambda^p(E) \otimes S^q(E)$  into a direct sum of irreducible representations of  $GL(E)$ .

**4.** Let  $p$  be a prime number and  $\mathbb{F}_p$  be the field with  $p$  elements. Classify irreducible and projective indecomposable representations of  $S_p$  over  $\mathbb{F}_p$ .

**5.** Classify indecomposable representation of a quiver  $E_6$ . You can choose any orientation.

**6.** Let  $\text{char } k > 0$  and  $G$  be a finite group. Show that any projective  $k(G)$ -module is injective and that any projective indecomposable  $k(G)$ -module has a unique simple submodule.