

**Sample midterm.**

1. Find

$$\lim_{x \rightarrow 0} x\Gamma(x).$$

2. Express the integral in terms of special functions

$$\int_0^5 x^{-1/3}(5-x)^{10/3} dx$$

3. Find the circumference of an ellipse with half axes 2 and 1 in term of elliptic integrals.

4. Solve the differential equation

$$x^2 y'' - 5xy' + 8y = 0$$

using the power series method.

5. Prove that

$$\sum_0^n (2l+1)P_l(x) = P'_n(x) + P'_{n+1}(x).$$

6. Let  $f(x) = \sum_{l=0}^{\infty} c_l P_l(x)$  be the Legendre series. Find  $\int_{-1}^1 x f(x) dx$ .