

PRACTICE MIDTERM # 2
MATH 121B

1. Solve the differential equation

$$xy'' + (1 - x)y' + y = 0$$

Hint: one solution is polynomial.

2. (a) Using the generating function

$$\Phi(x, h) = e^{2xh - h^2} = \sum_{n=0}^{\infty} H_n(x) \frac{h^n}{n!},$$

prove the recursion relation of the Hermite polynomials

$$H_{n+1}(x) = 2xH_n(x) - 2nH_{n-1}(x).$$

(b) Evaluate $H_n(0)$ for all n .

3. A string of length 3 has a 0 initial velocity; the initial displacement is given by the function

$$f(x) = \begin{cases} 0.1x & \text{if } 0 \leq x \leq 1, \\ 0.15 - 0.05x & \text{if } 1 \leq x \leq 3. \end{cases}$$

Assuming the wave velocity $v = 1$, find the position of the midpoint of the string at $t = 1, 2, 3$.

4. Find the steady-state temperature in a square plate of size 10 by 10 if the top and bottom sides are kept at temperature 50° , and right and left sides are held at 0° .