PROBLEM SET # 13
MATH 114

Due May 4.

1. Prove that the Galois group of $f(x) = x^5 + x^4 - 4x^3 - 3x^2 + 3x + 1$ over $\mathbb{Q}$ is cyclic of order 5. Hint: let ω be 11-th root of 1. Prove that $f(x)$ is the minimal polynomial for $\omega + \omega^{-1}$.

2. Let p be an odd prime, ω be a primitive p-th root of 1.

(a) Prove that $\mathbb{Q}(\omega)$ contains exactly one quadratic extension of $\mathbb{Q}$;

(b) If $p = 4k + 1$, then this quadratic extension is isomorphic to $\mathbb{Q}(\sqrt{p})$;

(c)If $p = 4k + 3$, then this quadratic extension is isomorphic to $\mathbb{Q}(\sqrt{-p})$.

3. Find the Galois group of $x^4 + 2x^3 + x + 3$ over $\mathbb{Q}$ using reduction modulo 2 and 3.

4. Give an example of a polynomial of degree 6 whose Galois group over $\mathbb{Q}$ is S_6.

Date: April 27, 2006.