PROBLEM SET # 7
MATH 114

Due March 16.

1. Find the Galois groups of the following polynomials over $\mathbb{Q}$:

$$(a) x^4 + x^2 + 1;$$

$$(b)(x^2 - 2)(x^2 - 3)(x^2 - 5);$$

$$(c) x^6 - 3;$$

$$(d) x^5 - 2.$$

2. Which of the following are normal extensions?

$$(a)\mathbb{Q} \subset \mathbb{Q}[x]/(x^3 + x + 1);$$

$$(b)\mathbb{Z}_2 \subset \mathbb{Z}_2[x]/(x^3 + x + 1);$$

$$(c)\mathbb{Q} \subset \mathbb{Q}[x]/(x^4 + 25).$$

3. Let $F \subset E$ be a normal extension and (E/F) be prime. What is $Aut_F E$?

4. Let $f(x)$ be an irreducible polynomial over $\mathbb{Q}$ of prime degree and $f(x)$ have exactly two complex roots. Prove that the Galois group of $f(x)$ is isomorphic to S_p. Hint: check that the Galois group contains a p-cycle and a transposition.

Date: March 9, 2006.