PROBLEM SET # 6
MATH 114

Due March 2.

1. Let F be a field. Prove that every extension $F \subset E$ of degree 2 is isomorphic to $F\left(\sqrt{d}\right)$ for some $d \in F$ which is not a perfect square in F.

2. Find the degree of the splitting field of the polynomial $x^3 + x + 1 \in \mathbb{Q}[x]$. Hint: check that the polynomial is irreducible and has only one real root.

3. Find the degree of the splitting field of the polynomial $x^3 + x + 1 \in \mathbb{Z}_2[x]$. Hint: check that if α is a root, then α^2 is also a root.

4. Describe the groups of automorphisms for the splitting fields in problems 2 and 3.

A character of a group G in a field F is a map $\sigma\colon G \to F$ such that $\sigma(gh) = \sigma(g)\sigma(h)$ for any $g, h \in G$ (see page 34 in Artin).

5. Find all characters of S_n in $\mathbb{C}$.

6. Let G be a cyclic group of order n. Find all characters of G in $\mathbb{R}$ and in $\mathbb{C}$.

7. Let G be a finite abelian group. Prove that the number of characters of G in $\mathbb{C}$ equals the order of G.

Date: February 22, 2006.