PROBLEM SET # 5
MATH 114

Due February 23.

1. Determine which of the following polynomials are irreducible over $\mathbb{Q}$:

$$x^4 + x^2 + 1,\ x^{12} + 99,\ x^3 + 2x + 1.$$

2. List all irreducible polynomials of degree 4 in $\mathbb{Z}_2[x]$.

3. Let F be a field, $f(x), g(x) \in F[x]$ and $f(\alpha) = g(\alpha)$ for any $\alpha \in F$. Prove that if F is infinite then $f(x) = g(x)$. Show that if F is finite, then the statement is wrong.

4. Let p be a prime number. Prove that $f(x) = x^{p-1} + x^{p-2} + \cdots + 1$ is irreducible over $\mathbb{Q}$. Hint: first check that $f(x)$ is irreducible if and only if $f(x+1)$ is irreducible. Use $f(x) = \frac{x^p - 1}{x - 1}$. Prove that $f(x+1)$ is irreducible by Eisenstein criterion.

5. Find $\left(\mathbb{Q}\left(\sqrt[3]{7}, \sqrt[17]{22}\right)/\mathbb{Q}\right)$.

6. Check that $\mathbb{Z}_{11}\left(\sqrt{2}\right)$ and $\mathbb{Z}_{11}\left(\sqrt{7}\right)$ are isomorphic.

7. Find the minimal polynomial for $\sqrt{7} + \sqrt{3}$ over $\mathbb{Q}$.

Date: February 20, 2006.