PROBLEM SET # 3
MATH 114

Due February 9.

1. Read notes on Sylow theorems. Prove the last corollary in these notes.

2. If p is prime and p divides $|G|$, then G has an element of order p.

3. Let p and q be prime and $q \not\equiv 1 \mod p$. If $|G| = p^n q$, then G is solvable.

4. Suppose that $|G| < 60$ and $|G| = 2^m 3^n$. Check that G is solvable. Hint: prove by induction on $|G|$. First, show that the number of Sylow 2-subgroups is 3 or the number of Sylow 3-subgroups is 4. Then construct a homomorphism $f : G \to S_3$ or S_4. By induction the kernel and the image of f are solvable. Hence G is solvable.

5. Show that any group of order less than 60 is solvable. Hint: use the previous problems to eliminate most of numbers below 60.

6. Let H be a p-subgroup of G, in other words $|H|$ is a power of a prime p. Prove that there is a Sylow p-subgroup P containing H. Hint: consider the action of H on the set of all Sylow p-subgroups. Check that there is a 1-element H-orbit $\{P\}$. Prove that H is a subgroup of P.

7. List all non-isomorphic abelian groups of order 60.

Date: February 1, 2006.