PROBLEM SET # 11
MATH 114

Due April 20.

1. Let E and B be normal extensions of F and $E \cap B = F$. Prove that

$$\operatorname{Aut}_F EB \cong \operatorname{Aut}_F E \times \operatorname{Aut}_F B.$$

2. Find the Galois group of the polynomial $(x^3 - 3)(x^3 - 2)$ over $\mathbb{Q}$.

3. Let $f(x)$ be an irreducible polynomial of degree 7 solvable in radicals. List all possible Galois groups for $f(x)$.

4. Find the Galois group of $x^6 - 4x^3 + 1$ over $\mathbb{Q}$.

5. Let $f(x) = g(x)\,h(x)$ be a product of two irreducible polynomials over a finite field $\mathbb{F}_q$. Let m be the degree of $g(x)$ and n be the degree of $h(x)$. Show that the degree of the splitting field of $f(x)$ over $\mathbb{F}_q$ is equal to the least common multiple of m and n.

6. Let $F \subset E$ be a normal extension with Galois group isomorphic to $\mathbb{Z}_2 \times \cdots \times \mathbb{Z}_2$. Assuming that $\operatorname{char} F \neq 2$, prove that

$$E = F\left(\sqrt{b_1}, \ldots, \sqrt{b_s}\right)$$

for some $b_1, \ldots, b_s \in F$.

7. Prove that the splitting field of the polynomial $x^4 + 3x^2 + 1$ over $\mathbb{Q}$ is isomorphic to $\mathbb{Q}\left(i, \sqrt{5}\right)$.

Date: April 12, 2006.