PROBLEM SET # 1
MATH 114

Due January 26.

1. Prove that a subgroup of a cyclic group is cyclic. (You have to consider both infinite and finite cyclic group).

2. Let G be a group and the order of G be even. Show that there is $a \in G$ of order 2. Hint if $a^2 \neq 1$, then $a \neq a^{-1}$.

3. Let $\mathbb{Q}$ be the set of all rational numbers. Consider $\mathbb{Q}$ as an abelian group with operation of addition. Show that $\mathbb{Q}$ is not cyclic.

4. Let D_4 denote the group of symmetries of a square. Find the order of D_4 and list all normal subgroups in D_4.

5. Let G be a group. By $Z(G)$ we denote the center of G, which is by definition, the set of all elements $g \in G$ such that $gx = xg$ for all $x \in G$. Assume that $G/Z(G)$ is a cyclic group. Prove that $Z(G) = G$, i.e. G is abelian.

6. Show that the n-cycle $(1 \ldots \mathrm{n})$ and the transposition (12) generate the permutation group S_n, i.e. every element of S_n can be written as a product of these elements.

7. Find a cyclic subgroup of maximal order in S_8.

Date: January 19, 2006.