

**PROBLEM SET # 12**  
**MATH 114**

Due April 27.

1. Trisect the angle of  $18^\circ$  by ruler and compass.
2. Let  $l$  be the least common multiple of  $m$  and  $n$ . Assume that regular  $m$ -gon and regular  $n$ -gon are constructible by ruler and compass. Prove that regular  $l$ -gon is also constructible by ruler and compass.
3. Find the minimal equation over  $\mathbb{Q}$  for  $\cos \frac{2\pi}{7}$ . Hint: express  $\cos \frac{2\pi}{7}$  in terms of 7-th roots of 1.
4.
  - (a) Prove that the angle of  $25^\circ$  is not constructible by ruler and compass;
  - (b) Prove that angle of  $n^\circ$  is constructible by ruler and compass if and only if  $n$  is a multiple of 3.
5. Given three segments  $a, b$  and  $c$ , construct a triangle whose altitudes equal  $a, b$  and  $c$ .
6. Let  $f(x)$  be an irreducible polynomial over  $\mathbb{Q}$  of degree 7. Assume that  $f(x)$  has exactly three real roots. Prove that  $f(x)$  is not solvable in radicals.