PROBLEM SET # 5
MATH 114

Due April 13.

1. Find the Galois group of the polynomial $x^4 + 8x + 12$ over $\mathbb{Q}$.

2. Find the Galois group of the polynomial $x^4 + 3x + 3$ over $\mathbb{Q}$.

3. Find the Galois group of $x^6 - 3x^2 + 1$ over $\mathbb{Q}$.

4. Assume that a polynomial $x^4 + ax^2 + b \in \mathbb{Q}[x]$ is irreducible. Prove that

(a) The Galois group is the Klein subgroup of S_4 if and only if $\sqrt{b} \in \mathbb{Q}$;

(b) The Galois group is a cyclic subgroup of S_4 if and only if $\sqrt{b}\sqrt{a^2 - 4b} \in \mathbb{Q}$;

(c) The Galois group is isomorphic to D_4 if and only if $\sqrt{b}, \sqrt{b}\sqrt{a^2 - 4b} \notin \mathbb{Q}$.

5. Let $f(x)$ be an irreducible polynomial of degree 5. List all (up to an isomorphism) subgroups of S_5 which can be the Galois group of $f(x)$. For each group G in your list give an example of an irreducible polynomial of degree 5 whose Galois group is G.

6. Let G be an arbitrary finite group. Show that there is a field F and a polynomial $f(x) \in F[x]$ such that the Galois group of $f(x)$ is isomorphic to G. Hint: use Theorem 6 on page 76 in the book.

Date: April 6, 2006.