QUALIFYING EXAM SYLLABUS

SCOTT CRAMER

Committee: Hugh Woodin, John Steel, Donald Sarason (Chair), Sherri Roush (Philosophy)

1. Combinatorial Set Theory and Forcing (Major Topic: Foundations)

Rank, trees, posets. Clubs, stationary sets, Fodor's Lemma. Mostowski collapse. Reflection. Partition properties, Erdös-Rado Theorem. MA, $\diamondsuit$, $\square$, Kurepa, Aronszajn, Suslin trees and Suslin lines. Δ-system lemma. Basic forcing constructions, The Generic Model Theorem. Preservation of cardinality, cofinality: chain conditions, homogeneity. Levy collapse. $\neg$CH. Product, Iterated forcing. Consistency of MA + $\neg$CH. Levy-Solovay Theorem: small forcing does not create or destroy large cardinals. Easton Forcing. Consistency of $\neg$AC. Prikry Forcing.

2. Large Cardinals and Inner Models (Major Topic: Foundations)

Inaccessible, Mahlo, Ramsey cardinals. Extenders. Measurable, strong, Woodin, superstrong, supercompact and huge cardinals. AD $\vdash \omega_1$ is measurable. Construction of L. $L \models ZFC + GCH + \diamondsuit + \Delta^1_2$-well ordering of $\mathbb{R}$. Condensation. Silver indiscernibles and sharps. Ultrapowers, $L[U]$, GCH in $L[U]$, Uniqueness of $L[U]$, Δ^1_3 -well ordering of $\mathbb{R}$. Vopênka's Theorem. Kunen's Theorem. $HOD \models ZFC$. Schoenfield Absoluteness.

3. Measure Theory (Minor Topic: Classical Analysis)

σ-algebras, outer measures, Carathéodory's Theorem, premeasures. Lebesgue measure. Borel sets. A set of reals which is not Lebesgue measurable. Measurable functions. Simple functions. Integration. Fatou's lemma. Monotone, Dominated Convergence Theorems. Product Measures. Fubini-Tonelli Theorem. Signed measures. Radon-Nikodym theorem. Hausdorff measure, Hausdorff dimension. Cantor set, Sierpinski triangle, Koch curve. Space filling curves.

Reference: Stein and Shakarchi, *Real Analysis*, Chapters 1, 2, 6, 7.