

Solutions to Quiz 3

Math 110

1. (a) True: there are elementary matrices E, E' such that $r(A) = EA$ and $c(A) = AE'$ for any A . Therefore, by associativity of matrix multiplication, we get

$$r(c(A)) = E(AE') = (EA)E' = c(r(A)).$$

- (b) True: If $Ax = Ay = b$, then $A(\frac{1}{3}x + \frac{2}{3}y) = \frac{1}{3}(Ax) + \frac{2}{3}(Ay) = \frac{1}{3}b + \frac{2}{3}b = b$.

(Note: this fits into the pattern of theorem 3.9 from the book as follows. We have $A(x - y) = 0$, so $x - y \in K_H$, and x is a particular solution. However, $\frac{1}{3}x + \frac{2}{3}y = x - \frac{2}{3}(x - y)$.)

2. Since A has rank 1, A has at least one nonzero column, and every other column of A is a scalar multiple of it. Thus, let X be one of the nonzero columns, and for each j find the scalar c_j such that the j th column of A is $c_j \cdot X$. Then we have

$$A = \begin{bmatrix} | & | & \cdots & | \\ c_1 X & c_2 X & \cdots & c_n X \\ | & | & & | \end{bmatrix} = X [c_1 \quad c_2 \quad \cdots \quad c_n].$$