Homework 4

Math 110

Due Tuesday, July 5

Section 2.3: #3, 9, 12, 13, 16

X1. Let $A \in M_{m\times n}(F)$ for some field F. Prove that the range (or image) of $L_A : F^n \to F^m$ is the span of the columns of A.

X2. (a) For $w \in \mathbb{C}$, let $M_w : \mathbb{C} \to \mathbb{C}$ be given by $T(z) = w \cdot z$. Consider $\mathbb{C}$ as a vector space over $\mathbb{R}$ with the standard ordered basis $\beta = \{1, i\}$. If $w = a + bi$ for real numbers a, b, calculate $[M_w]_\beta$.

(b) Let $w_1, w_2 \in \mathbb{C}$. Show that $M_{w_1} + M_{w_2} = M_{w_1+w_2}$, and $M_{w_1} \circ M_{w_2} = M_{w_1 w_2}$.

(c) Let $w_1, w_2 \in \mathbb{C}$. Verify by direct calculation that $[M_{w_1}]_\beta + [M_{w_2}]_\beta = [M_{w_1+w_2}]_\beta$ and $[M_{w_1}]_\beta [M_{w_2}]_\beta = [M_{w_1 w_2}]_\beta$.

(This gives a "representation" of $\mathbb{C}$ in terms of real matrices.)

X3. For two $n \times n$ matrices A, B, we define their *commutator* to be $[A, B] = AB - BA$.

Let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix},$$

and consider the function $T : M_{2\times 2}(\mathbb{R}) \to M_{2\times 2}(\mathbb{R})$ defined by $T(B) = [A, B]$.

(a) Show that T is linear.

(b) Find bases for $N(T) = \ker T$ and $R(T) = \operatorname{im} T$.

Section 2.4: #9, 17(b)
Section 2.5: #4, 7, 10