# Section 7.6: Nonlinear regression

**Problem**: For a given class of functions $\mathcal{F}$ and a set of data $\{(x_1, y_1), \ldots, (x_n, y_n)\}$ find a (the?) function $f$ of class $\mathcal{F}$ which minimizes the sum of squares of errors.

*Linear regression* is the solution to this problem where $\mathcal{F}$ is the class of linear functions.



# Quadratic regression

A *quadratic function* is a function $f(x)$ of the form $f(x) = ax^2 + bx + c$ for fixed constants $a$, $b$, and $c$.

Note that taking $a = 0$ we see that a linear function is a special case of a quadratic function.

Large An example of quadratic regression

Find a quadratic function $f(x) = ax^2 + bx + c$ minimizing the sum of squares of errors for the data $\{(-1,0),(1,2),(0,-1),(2,1)\}$.



# Solution

$$
\begin{aligned}
S(a,b,c) &= (f(-1)-0)^2 + (f(1)-2)^2 \\
&\quad +(f(0)-(-1))^2 + (f(2)-1)^2 \\
&= (a(-1)^2 + b(-1) + c - 0)^2 + (a(1)^2 + b(1) + c - 2)^2 \\
&\quad +(a(0)^2 + b(0) + c - (-1))^2 + (a(2)^2 + b(2) + c - 1)^2 \\
&= (a-b+c)^2 + (a+b+c-2)^2 \\
&\quad +(c+1)^2 + (4a+2b+c-1)^2
\end{aligned}
$$

## Solution, continued

$$\begin{aligned}
\frac{\partial S}{\partial a} &= 2[(a-b+c)+(a+b+c-2)+4(4a+2b+c-1)] \\
&= 2[18a+8b+6c-6] \\
\frac{\partial S}{\partial b} &= 2[(-1)(a-b+c)+(a+b+c-2)+2(4a+2b+c-1)] \\
&= 2[8a+6b+2c-4] \\
\frac{\partial S}{\partial c} &= 2[(a-b+c)+(a+b+c-2)+(c+1)+(4a+2b+c-1)] \\
&= 2[6a+2b+4c-2]
\end{aligned}$$

Setting these equal to zero and performing some algebra, we find $a = 0$, $b = \frac{3}{5}$, and $c = \frac{1}{5}$.



## Other forms of nonlinear regression

- Exponential regression: $f(x) = ae^{bx}$
- Power regression: $f(x) = ax^b$
- Logarithmic regression: $f(x) = a\ln(x) + b$
- Cubic regression: $f(x) = ax^3 + bx^2 + cx + d$
- Polynomial regression of degree $m$:
  $f(x) = a_m x^m + a_{m-1}x^{m-1} + \cdots + a_1 x + a_0 = \sum_{i=0}^{m} a_i x^i$

# Multiple regression

**Problem**: Given a set of data $\{(x_1, y_1, z_1), \ldots, (x_n, y_n, z_n)\}$ find a function $f(x, y)$ of two variables (of some specified class) best fitting the data.

When $f(x, y) = ax + by + c$ is taken to be a linear function, then this is the problem of *multiple linear regression*.