Section 7.6: Nonlinear regression

Problem: For a given class of functions \mathcal{F} and a set of data $\{(x_1, y_1), \ldots, (x_n, y_n)\}$ find a (the?) function f of class \mathcal{F} which minimizes the sum of squares of errors.

Linear regression is the solution to this problem where ${\mathcal F}$ is the class of linear functions.

1

Quadratic regression

A quadratic function is a function f(x) of the form $f(x) = ax^2 + bx + c$ for fixed constants a, b, and c.

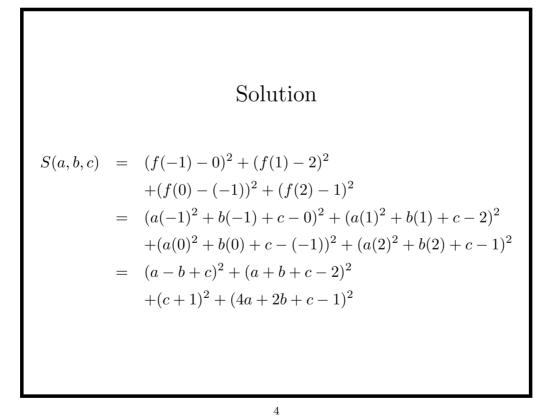
Note that taking a = 0 we see that a linear function is a special case of a quadratic function.

 $\mathbf{2}$

Large An example of quadratic regression

Find a quadratic function $f(x) = ax^2 + bx + c$ minimizing the sum of squares of errors for the data $\{(-1,0), (1,2), (0,-1), (2,1)\}$.

3



Solution, continued

$$\begin{array}{rcl} \frac{\partial S}{\partial a} &=& 2[(a-b+c)+(a+b+c-2)+4(4a+2b+c-1)]\\ &=& 2[18a+8b+6c-6]\\\\ \frac{\partial S}{\partial b} &=& 2[(-1)(a-b+c)+(a+b+c-2)+2(4a+2b+c-1)]\\ &=& 2[8a+6b+2c-4]\\\\ \frac{\partial S}{\partial c} &=& 2[(a-b+c)+(a+b+c-2)+(c+1)+(4a+2b+c-1)]\\ &=& 2[6a+2b+4c-2] \end{array}$$

Setting these equal to zero and performing some algebra, we find $a = 0, b = \frac{3}{5}$, and $c = \frac{1}{5}$.

5



- Exponential regression: $f(x) = ae^{bx}$
- Power regression: $f(x) = ax^b$
- Logarithmic regression: $f(x) = a \ln(x) + b$
- Cubic regression: $f(x) = ax^3 + bx^2 + cx + d$
- Polynomial regression of degree m: $f(x) = a_m x^m + a_{m-1} x^{m-1} + \dots + a_1 x + a_0 = \sum_{i=0}^m a_i x^i$

6

Multiple regression

Problem: Given a set of data $\{(x_1, y_1, z_1), \ldots, (x_n, y_n, z_n)\}$ find a function f(x, y) of two variables (of some specified class) best fitting the data.

When f(x, y) = ax + by + c is taken to be a linear function, then this is the problem of *multiple linear regression*.

 $\overline{7}$