Other trigonometric functions

$$\begin{aligned}
\tan(\theta) &:= \frac{\sin(\theta)}{\cos(\theta)} \\[1em]
\cot(\theta) &:= \frac{\cos(\theta)}{\sin(\theta)} \\[1em]
\sec(\theta) &:= \frac{1}{\cos(\theta)} \\[1em]
\csc(\theta) &:= \frac{1}{\sin(\theta)}
\end{aligned}$$

Derivative of the tangent function

$$\begin{aligned}
\frac{d}{dt}(\tan t) &= \frac{d}{dt}\left(\frac{\sin t}{\cos t}\right) \\[1em]
&= \frac{\left(\frac{d}{dt}(\sin t)\right)\cos t - (\sin t)\frac{d}{dt}(\cos t)}{\cos^2 t} \\[1em]
&= \frac{\cos^2 t + \sin^2 t}{\cos^2 t} \\[1em]
&= \frac{1}{\cos^2 t} \\[1em]
&= \sec^2 t
\end{aligned}$$

Derivative of the secant function

$$\frac{d}{dt}(\sec t) \quad = \quad \frac{d}{dt}\left(\frac{1}{\cos t}\right)$$

$$= \quad \frac{-1}{\cos^2 t}\frac{d}{dt}(\cos t)$$

$$= \quad \frac{\sin t}{\cos^2 t}$$

$$= \quad (\tan t)(\sec t)$$

Differentiating functions built from trigonometric functions

Let $g(t) = \tan(\ln(\sec t))$. Compute $g'(t)$.

Solution

$$
\begin{aligned}
g'(t) &= \frac{d}{dx}(\tan x)|_{x=\ln(\sec t)} \frac{d}{dt}(\ln(\sec t)) \\
&= \sec^2(\ln(\sec t)) \frac{d}{dx}(\ln x)|_{x=\sec t} \frac{d}{dt}(\sec t) \\
&= \sec^2(\ln(\sec t)) \frac{1}{\sec t}(\tan t)(\sec t) \\
&= \sec^2(\ln(\sec t)) \tan t
\end{aligned}
$$

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