

## Math 53 - Midterm 1 Review

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1. Eliminate the parameter to find a cartesian equation of the curve given by the parametric equations  $x = 2 \sinh t$  and  $y = 3 \cosh t$ .
2. Find parametric equations for the curve of intersection of the surfaces  $y = z^2 - x^2$  and  $z = 3 - x^2$ . Are there any points on the curve with zero velocity?
3. Sketch the curve given by the polar equation  $r = 1 + 2 \cos \theta$  and find the area of the region inside the outer loop but outside the inner loop.
4. Suppose that  $|\mathbf{a}| = 4$ ,  $|\mathbf{b}| = 2$ , and  $\mathbf{a} \cdot \mathbf{b} = 5$ . Compute the area of the parallelogram spanned by  $\mathbf{a}$  and  $\mathbf{b}$ .
5. Find the equation of the plane containing the points  $(1, 0, -2)$ ,  $(1, 1, 1)$ , and  $(-2, 3, 1)$ .
6. State the relations between rectangular and cylindrical coordinates, and between rectangular and spherical coordinates.
7. Identify each of the following surfaces: (a)  $\theta = \pi/4$ , (b)  $\phi = \pi/4$ , (c)  $x^2 + 2x - y^2 + 4y + z^2 = 0$ .
8. Show that the curve  $x = \sqrt{6} \cos t - 1$ ,  $y = 5$ ,  $z = \sqrt{6} \sin t$  lies on the surface  $x^2 + 2x - y^2 + 4y + z^2 = 0$ .
9. Let  $f(x, y) = 1 - \sqrt{x^2 + y^2}$ . Show that  $f$  satisfies the partial differential equation  $yu_x - xu_y = 0$ . Draw a contour map of  $f$  and describe its graph.
10. Find the tangent plane to the graph of  $f(x, y) = e^{-x^2}(y - \sin(x - y))$  at  $(0, 0)$ . Use this to approximate the value of  $f(.01, -.01)$ .
11. Compute the following limit, or show it doesn't exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x+y}{\sqrt{x^2+y^2}}$$