Practice Problems: Simple Statements about Sets

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Assume that A, B, and C are subsets of a set X. Some of the following statements are true, and others are false. Prove the statements which are true, and find counterexamples to the statements which are false.

1. If A and B are disjoint if and only if $A \subseteq (X \backslash B)$.
2. If $A \cap D \subseteq B \cap D$ for every $D \subseteq X$, then $A \subseteq B$.
3. The set $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$.
4. The set $X \backslash (A \cup B) = (X \backslash A) \cap (X \backslash B)$.
5. The set $X \backslash (A \cap B) = (X \backslash A) \cap (X \backslash B)$ if and only if $A = B$.
6. The set $A \subseteq B$ if and only if $X \backslash B \subseteq X \backslash A$.
7. The set $A \backslash B = (X \backslash B) \backslash (X \backslash A)$.
8. The set $A \backslash B = (X \backslash A) \backslash (X \backslash B)$ if and only if $A = B$.
9. The set $(A \cup B) \cap C = A \cup (B \cap C)$ if and only if $A \subseteq C$.
10. If $A \subseteq B$, then $C \backslash A \subseteq C \backslash B$ if and only if $B \backslash A$ and C are disjoint.
11. The set $A \cap B = A \backslash B$ if and only if $A = \varnothing$.
12. If $A \cap B \subseteq C$ and $A \cup B \subseteq (C \cup (A \cap B))$, then A and B are subsets of C.