Practice Problems: Negating Statements (SOLUTIONS)

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1. For every $M > 0$ there exists $x \in \mathbb{R}$ such that $|f(x)| > M$.

2. There exists $\varepsilon > 0$ such that for every positive integer N there exists an integer $n \geq N$ such that $|x_n - 3| \geq \varepsilon$.

3. For every real number c there exists $\varepsilon > 0$ such that for all positive integers N there exists a positive integer $n \geq N$ such that
$$|x_n - c| \geq \varepsilon.$$

4. For every function $f : \mathbb{R} \to \mathbb{R}$ there exists $x_0 \in \mathbb{R}$ and $\varepsilon > 0$ such that for every positive integer N there exists $n \geq N$ such that
$$|f_n(x_0) - f(x_0)| \geq \epsilon.$$

5. For every function $f : \mathbb{R} \to \mathbb{R}$ there exists $\varepsilon > 0$ such that for every positive integer N there exists an integer $n \geq N$ and a real number$x \in \mathbb{R}$ such that
$$|f_n(x) - f(x)| \geq \epsilon.$$

6. There exists $\varepsilon > 0$ such that for all $\delta > 0$ there exists $x \in \mathbb{R}$ such that $|x - x_0| < \delta$ and $|f(x) - f(x_0)| \geq \varepsilon$.

7. There exists $x_0 \in \mathbb{R}$ and $\varepsilon > 0$ such that for all $\delta > 0$ there exists $x \in \mathbb{R}$ such that $|x - x_0| < \delta$ and $|f(x) - f(x_0)| \geq \varepsilon$.

8. There exists $\varepsilon > 0$ such that for all $\delta > 0$ there exists $x, y \in \mathbb{R}$ such that $|x - y| < \delta$ and $|f(x) - f(y)| \geq \varepsilon$.

9. There exist vectors $v, w \in V$ and constants $a, b \in \mathbb{R}$ such that
$$T(av + bw) \neq aT(v) + bT(w).$$

10. For every $M > 0$ there exists $\alpha \in \Lambda$ and $x \in X$ such that
$$\|T_\alpha(x)\| > M\|x\|.$$

11. For every $\gamma < 1$ there exists $x, y \in \mathbb{R}$ such that
$$|f(x) - f(y)| > \gamma|x - y|$$

12. There exist constants $a_1, a_2, \ldots, a_k \in \mathbb{R}$ such that $a_j \neq 0$ for some j and
$$\sum_{j=1}^{k} a_j v_j = 0.$$

13. For every real number c there exists $\varepsilon > 0$ such that for all $\delta > 0$ there exists $x \in \mathbb{R}$ such that $0 < |x - x| < \delta$ and
$$\left| \frac{f(x) - f(x_0)}{x - x_0} - c \right| \geq \varepsilon.$$

14. There exists $x \in U$ such that for every $\varepsilon > 0$ there exists $y \in \mathbb{R} \backslash U$ such that $|x - y| < \varepsilon$.

15. There exists $\varepsilon > 0$ such that $|x - y| \geq \varepsilon$ for all $x \in U$.

16. For every constant L there exists $x, y \in \mathbb{R}$ such that
$$|f(x) - f(y)| > L|x - y|.$$

17. For each real number $p > 0$ there exists $x \in \mathbb{R}$ such that $f(x+p) \neq f(x)$.

18. There exists $x \in \mathbb{R}$ and $\varepsilon > 0$ such that $|x - y| \geq \varepsilon$ for all $y \in D$.

19. (Note: in this problem there was a typo– the phrase "$y \notin W$" should be replaced with "$z \notin W$.) There exists $x \in \mathbb{R}$ and $\varepsilon > 0$ such that for every $y \in \mathbb{R}$ satisfying $|x - y| < \varepsilon$ and every $\delta > 0$, there exists $z \in W$ such that $|y - z| < \delta$.