

Name: Solutions

## Quiz 7

Math 74

November 29, 2006

1. Does the system of linear congruences

$$\begin{cases} x \equiv 29 \pmod{75} \\ x \equiv 37 \pmod{177} \end{cases}$$

have any solutions? Why or why not?

No, since  $8 = 37 - 29$  isn't divisible  
by  $3 = \gcd(75, 177)$ .

2. Find one solution  $x \in \mathbb{Z}$  (if any) of the linear congruence

$$132x \equiv 231 \pmod{319}$$

We carry out the Ext. Alg.:

$$\begin{aligned} 319 \cdot 1 + 132 \cdot 0 &= 319 \\ 319 \cdot 0 + 132 \cdot 1 &= 132 \\ 319 \cdot 1 + 132 \cdot (-2) &= 55 \\ 319 \cdot (-2) + 132 \cdot 5 &= 22 \\ 319 \cdot (5) + 132 \cdot (-12) &= 11 \\ 319 \cdot (-12) + 132 \cdot 29 &= 0 \end{aligned}$$

$$\begin{aligned} \text{Hence } \gcd(319, 132) &= 11 \\ &= 319 \cdot 5 + 132 \cdot (-12) \end{aligned}$$

Thus, multiplying by  $21 = \frac{231}{11}$  we get

$$231 = 21 \cdot 11 = 21 \cdot (319 \cdot 5 + 132 \cdot (-12)), \text{ so}$$

$$231 = 319 \cdot (21 \cdot 5) + 132 \cdot (-12 \cdot 21)$$

$$\text{So } x = -12 \cdot 21 = -252$$

satisfies

$$132x \equiv 231 \pmod{319}$$

3. Now find all solutions  $x \in \mathbb{Z}$  of the linear congruence above.

Since  $319 / \gcd(132, 319) = 29$ , we have that the solution set is

$$\{x \in \mathbb{Z} \mid x \equiv -252 \pmod{29}\} = \{x \in \mathbb{Z} \mid x \equiv 9 \pmod{29}\}.$$