Name: Solutions

Quiz 3

Math 74
September 22, 2006

1. Developing the appropriate context, write down the definition of image.

Suppose that $f: X \to Y$ is a function, and $A \subseteq X$. Then the <u>image</u> of <u>A</u> <u>under</u> <u>f</u> is the set

$$f(A) = \{f(x) \mid x \in A\}.$$

2. Is the following statement true? Provide a proof or a counterexample. Suppose that X and Y are sets, $f: X \to Y$, and $A, B \subseteq X$. Then

$$f(A \cap B) = f(A) \cap f(B).$$

We will provide a counterexample. Consider the map $f: \{0,1\} \to \{0\}$, $x \mapsto 0$. Let $A = \{0\}$, $B = \{1\}$. Then $A \cap B = \phi$, so $f(A \cap B) = f(\emptyset) = \phi$. But $f(A) = f(B) = \{0\}$, so $f(A) \cap f(B) = \{0\} \neq \phi = f(A \cap B)$.

3. Is the following statement true? Provide a proof or a counterexample. Suppose that X and Y are sets, $f: X \to Y$, and $A, B \subseteq X$. Then

$$f(A \cup B) = f(A) \cup f(B).$$

<u>proof.</u> Suppose $y \in f(A \cup B)$. Then there exists $x \in A \cup B$ such that $y = f(x)$. If $x \in A$, then clearly $y \in f(A)$. If $x \in B$, then clearly $y \in f(B)$. Either way, $y \in f(A) \cup f(B)$. Hence $f(A \cup B) \subseteq f(A) \cup f(B)$.

Now suppose $y \in f(A) \cup f(B)$. If $y \in f(A)$, then $y = f(x)$ for some $x \in A$. If $y \in f(B)$, then $y = f(x)$ for some $x \in B$. Either way, $y = f(x)$ for some $x \in A \cup B$. Hence $y \in f(A \cup B)$. Thus $f(A) \cup f(B) \subseteq f(A \cup B)$.