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Quiz 2

Math 74
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1. Developing the appropriate context, write down the definition of **preimage**.

Let $f: X \to Y$, and $B \subseteq Y$. Then the preimage of B under f is the set

$$f^{-1}[B] = \{x \in X \mid f(x) \in B\}.$$

2. Prove or provide a counterexample to the statement that for all sets A, B and C,

$$(A \cap B) \cup (A \cap C) = A \cap (B \cup C).$$

Suppose $x \in [A \cap B] \cup [A \cap C]$. Then clearly $x \in A \cap B$ or $x \in A \cap C$. In both cases, $x \in A$ and $x \in B \cup C$. Thus $x \in A \cap (B \cup C)$. Therefore, $[A \cap B] \cup [A \cap C] \subseteq A \cap (B \cup C)$.

Now suppose that $y \in A \cap (B \cup C)$. Then $y \in A$, and $y \in B$ or $y \in C$. In the first case, $y \in A \cap B$, and in the second case, $y \in A \cap C$. Either way, $y \in [A \cap B] \cup [A \cap C]$. Thus $A \cap (B \cup C) \subseteq (A \cap B) \cup (A \cap C)$.

3. Prove or provide a counterexample to the statement that for all sets A, B and C, if $A \cap B \subseteq A \cap C$, then $B \subseteq C$ if and only if $B \cup C \subseteq A$.

Let $A = B = \phi$, and $C = \{0\}$. Then $A \cap B = A \cap C = \phi$, and so $A \cap B \subseteq A \cap C$. Also, $B \subseteq C$, but $B \cup C \nsubseteq A$. Thus we have shown that there exist sets A, B, C, such that $A \cap B \subseteq A \cap C$, and $B \subseteq C$, but $B \cup C \nsubseteq A$.