Math 74 - (SAMPLE) Midterm #2

Monday, December 4, 2006, 3:00pm-4:00pm

Name: ______________________________

This is a closed book, closed notes exam. Calculators are not allowed. You have fifty minutes to complete the exam. To receive full credit, write legibly and in complete sentences. If you need more space, use the back of the page of the problem on which you are working.

Problem	Points	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	80	

1. (a) Prove that $\gcd(m, 0) = m$ for all integers $m > 0$.

(b) Prove that $\gcd(m, n) = \gcd(n, m \bmod n)$ for all positive integers m and n.

(c) Which important theorem, proved in class, do the above facts help to prove?

2. (a) State the Division Algorithm.

(b) Show that an integer n is not even if and only if there exists an integer k such that $n = 2k + 1$.

(c) Suppose that there exists an integer k such that $n = 12k + 5$. Must it be true that $\gcd(n, 12) = 1$?

3. (a) Find all solutions (if any) of the linear system

$$\begin{cases} x \equiv 44 \pmod{105} \\ x \equiv 17 \pmod{81}. \end{cases}$$

4. (a) Prove or provide a counterexample: For all positive integers m and n,

$$\gcd(m + 2n, 3n) = \gcd(m, n).$$

(b) Prove or provide a counterexample: If $\gcd(a, b) = 1$, then $\gcd(b, c) = 1$ if and only if $\gcd(a, c) = 1$.

5. (Bonus)

(a) Let a_n be the sequence recursively defined by

$$\begin{cases} a_1 = 2 \\ a_{k+1} = a_k^2 - a_k + 1 \ (k \geq 1) \end{cases}$$

Show that $a_n = 1 + \prod_{j=1}^{n-1} a_j$ for every n.

(b) Show that $\gcd(a_n, a_m) = 1$ for all positive integers m and n such that $m \neq n$.