Math 74 - (SAMPLE) Midterm #1

Friday, October 6, 2006, 3:00pm-4:00pm

Name: ____

This is a closed book, closed notes exam. Calculators are not allowed. You have fifty minutes to complete the exam. To receive full credit, write legibly and in complete sentences. If you need more space, use the back of the page of the problem on which you are working.

Problem	Points	Your Score
1	20	
2	20	
3	20	
4	20	
5	20	
Total	80	

1. (a) Write down the negation of the following statement: "For every $\epsilon > 0$ there exists a positive integer N such that $|x_m - x_n| \leq \epsilon$ for all positive integers m and n such that $m \geq N$ and $n \geq N$."

   (b) Give a proof or a counterexample to the following statement: Let A, B, and C be sets. Then $(A \backslash B) \backslash C = (A \backslash C) \backslash (B \backslash C)$.

2. (a) Prove the following statement or provide a counterexample: If $f : X \to Y$ is a map and $B_1, B_2 \subseteq Y$, then $B_1 \subseteq B_2$ if and only if $f^{-1}[B_1] \subseteq f^{-1}[B_2]$.

(b) Prove the following statement or provide a counterexample: If $f : X \to Y$ is a map, $A \subseteq X$ and $B \subseteq Y$, then $f(A) = B$ if and only if $f^{-1}[B] = A$.

3. (a) Developing the appropriate context, define the terms *injection*, *surjection* and *bijection*.

(b) Is the function $g : \mathbb{Z} \to \mathbb{Z}$ defined by $g(n) = 5 - 2n$ injective? Surjective? Bijective?

4. (a) State the Well-Ordering Principle.

(b) Suppose that $n \in \mathbb{N}^*$. Show that there exists a unique positive integer k such that $k(k-1) < n \leq k(k+1)$.

5. ((**Bonus Problem)** Suppose that $f : X \to Y$ and $g : Y \to Z$ such that the function $h : X \to Z$ given by $h(x) = g(f(x))$ is bijective.

   (a) Must f be injective? Surjective?

   (b) Must g be injective? Surjective?